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1 Rational Numbers

Revision of The Number Systems

Let us recall the number systems, that we have studied in our earlier classes. So far we have studied—

1. Natural Numbers
2. Whole Numbers
3. Fractional Numbers
4. Integers

NATURAL NUMBERS

The numbers other than zero are called natural numbers. Numbers like 1, 2, 3, 4, 5 .... are called natural numbers.

WHOLE NUMBERS

All the numbers used for counting including zero are called whole numbers. 0, 1, 2, 3, 4, 5 .... are whole numbers.

All whole numbers are natural numbers but all natural numbers are not whole numbers.

FRACTIONAL NUMBERS

The numbers in the form of $\frac{p}{q}$, whose p and q are whole numbers and $q \neq 0$ are fractional numbers.

The numbers $0, 1, 2\frac{1}{2}, \frac{2}{3}, \frac{12}{7}, 2\frac{1}{2}$ are fractional numbers.

INTEGERS

The numbers $-3, -2, -1, 0, 1, 2, 3$.......... are called integers.

The difference between fractional number and rational number : Fractional numbers include only positive integers whereas rational numbers include positive as well as negative integers.

Numbers like $\frac{3}{5}$ are fractional as well as rational numbers. Whereas $\frac{3}{-5}$ is a rational number but not a fraction.

Similarly all natural numbers are rational numbers also but all rational numbers are not natural numbers. All whole numbers are also rational numbers.

RATIONAL NUMBERS

All the numbers of the form of $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$. $-1, -2, -3, 0, 1, 2, 3,$ are rational $\frac{2}{3}, \frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}, \sqrt{4}, \sqrt{25}$ numbers.

Properties of Rational Numbers

1. Positive rational numbers : The rational numbers whose both the numerator and denominators are either positive or negative are said to be positive rational numbers.

2. Negative rational numbers : Rational numbers whose numerators or denominators are negative are said to be negative rational numbers or simply negative rationals.

3. Equivalent rational numbers : If $\frac{p}{q}$ is a rational number then $\frac{p}{q} = \frac{p \div m}{q \div m}$, where *m* is a non zero integer.

Example : $\frac{p}{q} = \frac{4}{16}, \frac{p}{q} = \frac{p \div m}{q \div m} = \frac{4 \div 2}{16 \div 2} = \frac{2}{8}$

4. If $\frac{p}{q}$ is a rational number and m is a common divisor of *p* and *q*. Then $\frac{p}{q} = \frac{p \div m}{q \div m}$. Where *m* is a non zero integer.

Example : $\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}$

5. Standard form of rational number : If $\frac{p}{q}$ is a rational number having no common divisor this rational number is said to be in the standard form.

The rational number $\frac{5}{7}$ is in standard form as it has no common divisor. A non standard rational number can be converted into standard form by dividing with a common divisor other than 1.

Example : Express $\frac{25}{45}$ in standard form.

Solution : $\frac{25}{45} = \frac{25 \div 5}{45 \div 5} = \frac{5}{9}$

$\frac{5}{9}$ is a rational number in the standard form as it has no more common divisor other than 1.

Comparison of Rational Numbers (Method – 1)

STEPS OF COMPARISON :

1. A rational number in the standard form must not have a negative denominator. If the denominator is negative convert it to positive.
2. Take LCM of all the denominators.
3. Work out the numerator as we do for addition and subtraction of fractional numbers.
4. Compare the numerators. The rational numbers having larger numerators are greater.

Example : For any given rational number $\frac{p}{q}$

Solution :

$$\left|\frac{p}{q}\right| = \begin{cases} \frac{p}{q} \text{ if } \frac{p}{q} > 0 \\ 0 \text{ if } \frac{p}{q} = 0 \\ -\frac{p}{q} \text{ if } \frac{p}{q} < 0 \end{cases}$$

for example, $\left|\frac{-3}{11}\right| = \left|\frac{3}{11}\right| \Rightarrow \left|\frac{-7}{-13}\right| = \frac{7}{13}$.

(Method – II)

Property : Let $\frac{a}{b}$ and $\frac{c}{d}$ be rational numbers where b and d are positive integers. Then.

$$\frac{a}{b} \times \frac{c}{d}$$

If $a \times d > c \times b$ then $\frac{a}{b} > \frac{c}{d}$

If $a \times d < c \times b$ then $\frac{a}{b} < \frac{c}{d}$

Example : Compare $\frac{-5}{7}$ and $-\frac{3}{4}$

Solution : $\frac{-5}{7} \times -\frac{3}{4}$

$-5 \times 4 = -20$

$-3 \times 7 = -21$

$-20 > -21$

Therefore $\frac{-5}{7} > -\frac{3}{4}$

Example : If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational number, then

Solution : $\frac{a}{b} x \frac{c}{d} = \frac{axc}{bxd} = \frac{\text{Product of numerators}}{\text{Product of denominators}}$

$\frac{2}{3} x \left(\frac{-7}{5}\right) = \frac{2x(-7)}{3x5} = \frac{-14}{15}$

$\frac{a}{b} x \frac{c}{d} x \frac{e}{f} x...... = \frac{axcxe.....}{bxdxf.....}$

Arranging Rational Numbers in Ascending and Descending Order

Example : Arrange $\frac{-2}{3}$, $\frac{-9}{15}$ and $\frac{-4}{5}$ in ascending order.

Solution : The LCM of denominators 3,15 and 5 is 15

$\frac{-2}{3} = \frac{-2 \times 5}{3 \times 5} = \frac{-10}{15}$

$\frac{-4}{5} = \frac{-4 \times 3}{5 \times 3} = \frac{-12}{15}$

$\frac{-9}{15} = \frac{-9 \times 1}{15 \times 1} = \frac{-9}{15}$

$\frac{-12}{15} < \frac{-10}{15} < \frac{-9}{15}$

$\therefore \frac{-4}{5} < \frac{-2}{3} < \frac{-9}{15}$

Exercise 1.1

1. **Express in standard form.**

(a) $\frac{4}{8}$ (b) $\frac{10}{30}$ (c) $\frac{11}{55}$ (d) $\frac{13}{65}$ (e) $\frac{24}{96}$

2. Write three equivalent rational numbers of $\frac{2}{9}$.

3. **Compare each pair of the given rational numbers.**

(a) $\frac{11}{25}, \frac{110}{250}$ (b) $\frac{6}{7}, \frac{36}{37}$ (c) $\frac{21}{57}, \frac{42}{114}$ (d) $\frac{5}{9}, \frac{100}{180}$ (e) $\frac{3}{7}, \frac{-3}{7}$

4. **Which of the following pairs of rational numbers are equal?**

(a) $\frac{-11}{7}, \frac{33}{-21}$ (b) $\frac{3}{-5}, \frac{6}{10}$ (c) $\frac{7}{4}, \frac{-28}{-16}$

(d) $\frac{3}{13}, \frac{-12}{52}$ (e) $\frac{4}{12}, \frac{-1}{3}$ (f) $\frac{2}{5}, \frac{5}{2}$

5. **Write each of the mixed fractions in p/q form.**

(a) $3\frac{4}{5}$ (b) $6\frac{2}{3}$ (c) $-5\frac{1}{4}$ (d) $-7\frac{2}{3}$

6. **Sort out the rational numbers which are not equal to $\frac{3}{5}$.**

(a) $\frac{-3}{5}$ (b) $\frac{3}{-5}$ (c) $\frac{3}{5}$ (d) $\frac{6}{10}$ (e) $\frac{30}{50}$

7. **Write rational numbers equivalent to $\frac{-3}{5}$ with denominators.**

(a) 20 (b) −30 (c) 35 (d) −40

8. **Fill in the blank boxes with symbols <, > or = .**

(a) $\frac{3}{8}$ ☐ 0 (b) $\frac{-2}{9}$ ☐ 0 (c) $\frac{-3}{4}$ ☐ $\frac{1}{4}$

(d) $\frac{-5}{7}$ ☐ $\frac{-4}{7}$ (e) $-\frac{2}{3}$ ☐ $-\frac{3}{4}$ (f) $\frac{-1}{2}$ ☐ 0

9. **Which of the two rational numbers is greater in the given pair.**

(a) $\frac{-12}{5}$ or -3 (b) $\frac{4}{-5}$ or $\frac{-7}{10}$ (c) $\frac{9}{-13}$ or $\frac{7}{-12}$

(d) $\frac{-1}{3}$ or $\frac{4}{-5}$ (e) $\frac{7}{-9}$ or $\frac{-5}{8}$ (f) $\frac{-4}{3}$ or $\frac{-8}{7}$

10. **Arrange in ascending order.**

(a) $\frac{4}{-9}, \frac{-5}{12}, \frac{7}{-18}, \frac{-2}{3}$ (b) $\frac{-3}{4}, \frac{5}{-12}, \frac{-7}{16}, \frac{9}{-24}$

(c) $\frac{3}{-5}, \frac{-7}{10}, \frac{-11}{15}, \frac{-13}{20}$ (d) $\frac{-4}{7}, \frac{-9}{14}, \frac{13}{-28}, \frac{-23}{42}$

11. **Arrange the following rational number in descending order.**

(a) $-2, \frac{-13}{6}, \frac{8}{-3}, \frac{1}{3}$ (b) $\frac{-3}{10}, \frac{7}{-15}, \frac{-11}{20}, \frac{17}{-30}$

(c) $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}, \frac{23}{-24}$ (d) $\frac{-10}{11}, \frac{-19}{22}, \frac{-23}{33}, \frac{-39}{44}$

12. **Find two rational numbers whose absolute value is $\frac{1}{5}$.**

13. Fill in the blank space –

(a) Every negative rational number is .... zero.

(b) If x, y, z are rational numbers such that $x > y$ and $y > z$ then ....

(c) Two rational numbers are said to be equal if they are equal in their .... form.

(d) If the integers p and q have no common divisor other than 1 and q is positive then the rational number is said to be in the .... form.

(e) If $\frac{p}{q}$ is a rational number, then q cannot be ....

(f) Between two rational numbers there lie .... number of rational numbers.

(g) The reciprocal of $\frac{1}{a}$, where $a \neq 0$ is ....

(h) The number which cannot be the reciprocal of any number is ....

(i) 1 and –1 are .... of itself.

(j) The product of a rational number and its reciprocal is ....

14. Mark (✓) for true or (×) for False.

(a) If $\frac{a}{b}$ is a rational number and m is an integer then $\frac{a}{b} = \frac{a \div m}{b \div m}$

(b) Every whole number is a rational number but every rational number is not a whole number.

(c) Zero is the smallest rational number.

(d) $\frac{a}{0}$ is rational number where $a \neq 0$.

(e) All integers are rational numbers.

(f) The quotient of two integers is always a rational number.

(g) The quotient of two integers is always an integer.

15. Encircle the correct answers.

(a) The greatest rational number out of the following rational numbers is ?

(i) $\frac{5}{-9}$ (ii) $\frac{5}{4}$ (iii) $\frac{5}{7}$ (iv) $\frac{-5}{6}$

(b) Which one is the smallest rational number?

(i) $\frac{3}{7}$ (ii) $\frac{4}{-7}$ (iii) $\frac{-5}{7}$ (iv) $\frac{2}{7}$

(c) Which of the following is not in standard form?

(i) $\frac{7}{5}$ (ii) $\frac{10}{20}$ (iii) $\frac{13}{33}$ (iv) $\frac{27}{28}$

(d) If $\frac{5}{8} = \frac{20}{x}$ then the value of x is –

(i) 23 (ii) –23 (iii) 32 (iv) 2

(e) If $\frac{1}{4}$ is written with denominator 12. Then its numerator will be –

(i) 48 (ii) 3 (iii) –8 (iv) 8

(f) Which of the following is a positive rational number ?

(i) $\frac{-3}{-4}$ (ii) $\frac{0}{4}$ (iii) $\frac{3}{-4}$ (iv) $\frac{-3}{4}$

Addition of Rational Numbers

PROPERTIES OF ADDITION OF RATIONAL NUMBERS

1. $\frac{a}{b}+\frac{c}{d}$ = A rational number – Closure property. The sum of two rational numbers is always a rational number.

Example 1 : Add $\frac{4}{9}$ and $\frac{-11}{9}$

Solution :

$$=\frac{4}{9}+\left(\frac{-11}{9}\right)=\frac{4+(-11)}{9}$$

$$=\frac{4-11}{9}=\frac{-7}{9}$$

2. $\frac{a}{b}+\frac{c}{d} = \frac{c}{d}+\frac{a}{b}$ Commutative property.

Example 2 : Add $\frac{1}{3}$ and $\frac{5}{6}$

Solution :

$$\frac{1}{3}+\frac{5}{6} = \frac{5}{6}+\frac{1}{3}$$

$$=\frac{2+5}{6} = \frac{2+5}{6}$$

$$=\frac{7}{6} = \frac{7}{6}$$

3. The, sum of two rational number is -2, if one of the numbers is $\frac{-14}{5}$, find the other.

Example 3 : Sum of two rational number of like -2, $\frac{-14}{5}$.

Solution : Let the number be x.

$$\Rightarrow x+\left(\frac{-14}{5}\right) = -2$$

$$\Rightarrow x = -2+\frac{14}{5}$$

$$=\frac{-10+14}{5}=\frac{4}{5}$$

Hence, the required number is $\frac{4}{5}$.

4. $\left(\frac{a}{b}+0\right) = \left(0+\frac{a}{b}\right) \quad \frac{a}{b} \quad =$ Associative property of zero.

That is when zero is added to any rational number the sum is the rational number itself.

Example 4 : Add $\frac{2}{5}$ and 0.

$$\left(\frac{2}{5}+0\right) = \left(0+\frac{2}{5}\right)$$

$$\left(\frac{2+0}{5}\right) = \left(\frac{0+2}{5}\right)$$

$$\frac{2}{5} = \frac{2}{5}$$

5. $\left(\frac{a}{b} + \frac{-a}{b}\right) = \left(\frac{-a}{b} + \frac{a}{b}\right) = 0$, Additive inverse.

For every rational number $\frac{a}{b}$ there exists a rational number $\frac{-a}{b}$ such that $\frac{a}{b} + \frac{-a}{b} = \frac{a-a}{b} = \frac{0}{b} = 0$

Therefore $\frac{-a}{b}$ and $\frac{a}{b}$ are additive inverse of each other.

Example 5 : Find additive inverse of $\frac{3}{7}$.

Solution : The additive inverse of $\frac{3}{7}$ is $\frac{-3}{7}$. It can be proved by adding it.

$$\left(\frac{3}{7} + \frac{-3}{7}\right) = \frac{3}{7} - \frac{3}{7} = \frac{3-3}{7} = \frac{0}{7} = 0$$

6. Subtraction of rational numbers.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers.

Then, $\frac{a}{b}$ + additive inverse of $\frac{c}{d} = \frac{a}{b} - \frac{c}{d}$ = rational number.

Example 1 : Find additive inverse of the following rational numbers.

(a) $\frac{3}{9}$ (b) $\frac{-17}{9}$ (c) $\frac{7}{-9}$ (d) $\frac{-4}{-9}$

Solution : (a) Additive inverse of $\frac{3}{9}$ is $\frac{-3}{9}$

(b) Additive inverse of $\frac{-17}{9}$ is $\frac{+17}{9}$ or $\frac{17}{9}$

(c) Additive inverse of $\frac{7}{-9}$

$\frac{7\times-1}{-9\times-1} = \frac{-7}{9}$ The additive inverse of $\frac{-7}{9}$ is $\frac{+7}{9}$ or $\frac{7}{9}$

(d) Additive inverse of $\frac{-4}{-9}$

$\frac{-4}{-9} = \frac{-4\times-1}{-9\times-1} = \frac{4}{9}$, The additive inverse of $\frac{4}{9}$ is $\frac{-4}{9}$.

Example 2 : Subtract $\frac{1}{4}$ from $\frac{2}{3}$.

Solution : $\left(\frac{2}{3}\right) - \left(\frac{1}{4}\right) = \frac{2}{3}$ + additive inverse of $\frac{1}{4}$

$$\frac{2}{3} + \left(\frac{-1}{4}\right) = \left(\frac{2}{3}\right) - \frac{1}{4}$$

$$\frac{8-3}{12} = \frac{5}{12}$$

Example 3 : Subtract $\frac{-3}{7}$ from $\frac{-2}{5}$

Solution : $\left(\frac{-2}{5} - \frac{-3}{7}\right) = \frac{-2}{5}$ + additive inverse of $\frac{-3}{7}$

$$= \frac{-2}{5} + \frac{3}{7}$$

$$= \frac{-14+15}{35} = \frac{1}{35}$$

Example 4 : What should be added to $\frac{-5}{8}$ to get $\frac{3}{9}$.

Solution : Let the number to be added be = x

$$\frac{-5}{8} + x = \frac{3}{9}$$

$$x = \frac{3}{9} + \frac{5}{8}$$

$$= \frac{24+45}{72}$$

$$= \frac{69}{72} \quad \text{Ans.}$$

Example 5 : The sum of two numbers is –7. If one of them is $\frac{-11}{6}$, find the other rational number.

Solution : Let the other number be = x

$$\frac{-11}{6} + x = -7$$

$$x = -7 + \frac{11}{6}$$

$$x = \frac{-42+11}{6} = \frac{-31}{6}$$

The other number is $\frac{-31}{6}$

Example 6 : Evaluate $\left(\frac{3}{5}\right)^3 \text{x} \left(\frac{3}{5}\right)^2$

The commutative property states that rational number can be arranged in desired way. The associative property states that rational number can be grouped in desired manner.

Solution :

$$= \left(\frac{3}{5}\right)^3 \text{x} \left(\frac{3}{5}\right)^2$$

$$= \left(\frac{3}{5}\right)^3 \text{x} \left(\frac{3}{5}\right)^2$$

$$= \left(\frac{3}{5}\right)^{3+2} \text{x} \left(\frac{3}{5}\right)^5$$

$$= \frac{3^5}{5^5} = \frac{243}{3125}$$

Example 7 : Simplify $\left(\frac{2}{3} + \frac{4}{7} + \frac{-8}{9} + \frac{-5}{21}\right)$

Solution :

$$\left[\frac{2}{3} + \frac{4}{7} + \left(\frac{-8}{9}\right) + \left(\frac{-5}{21}\right)\right]$$

$$\left[\frac{2}{3} + \left(\frac{-8}{9}\right)\right] + \left[\frac{4}{7} + \left(\frac{-5}{21}\right)\right]$$ – using commutative and associative identities.

$$\left(\frac{2}{3}-\frac{8}{9}\right)+\left(\frac{4}{7}-\frac{5}{21}\right)$$

$$=\left(\frac{6-8}{9}\right)+\left(\frac{12-5}{21}\right)$$

$$=\frac{-2}{9}+\frac{7}{21}=\frac{-14+21}{63}$$

$$=\frac{7}{63}=\frac{1}{9}$$

Example 8 : What should be subtracted from $\frac{-3}{7}$ to get '1'.

Solution : Let the number be added be = x

$$\frac{-3}{7}-x = 1$$

$$-x = 1+\frac{3}{7}$$

$$-x = \frac{7+3}{7}$$

$$-x = \frac{10}{7}$$

$$-x\times(-1) = \frac{10}{7}\times(-1)$$

$$x = \frac{-10}{7} \text{ ans.}$$

Example 9 : Find absolute values of the following rational numbers.

(a) $\frac{2}{7}$ (b) $\frac{-2}{7}$ (c) $\frac{21}{-9}$ (d) $\frac{-23}{27}$ (e) $\frac{-151}{309}$

Solution :

(a) $\left|\frac{2}{7}\right| = \frac{|2|}{|7|} = \frac{2}{7}$

(b) $\left|\frac{-2}{7}\right| = \frac{|-2|}{|7|} = \frac{2}{7}$

(c) $\left|\frac{21}{-9}\right| = \frac{|21|}{|-9|} = \frac{21}{9}$

(d) $\left|\frac{-23}{27}\right| = \frac{|-23|}{|27|} = \frac{23}{27}$

(e) $\left|\frac{-151}{309}\right| = \frac{|-151|}{|309|} = \frac{151}{309}$

Example 10: Add $\left|\frac{-3}{7}\right|$ and $\left|\frac{-9}{21}\right|$

Solution :

$$\left|\frac{-3}{7}\right|+\left|\frac{-9}{21}\right| = \frac{|-3|}{|7|}+\frac{|-9|}{|21|} = \frac{3}{7}+\frac{9}{21}$$

$$=\frac{9+9}{21}$$

$$=\frac{18}{21}=\frac{6}{7}$$

Exercise 1.2

1. Add the following –

(a) $\frac{4}{5}$ and $\frac{-2}{5}$ (b) $\frac{-4}{11}$ and $\frac{-6}{11}$ (c) $\frac{5}{6}$ and $\frac{-1}{6}$ (d) $\frac{-7}{3}$ and $\frac{1}{3}$ (e) $\frac{-17}{15}$ and $\frac{-1}{5}$

2. Find the sum of the following –

(a) $\frac{-3}{5}, \frac{3}{4}$ (b) $\frac{5}{8}, \frac{-7}{12}$ (c) $\frac{-8}{9}, \frac{11}{6}$

(d) $\frac{7}{24}, \frac{-5}{16}$ (e) $\frac{7}{-18}, \frac{8}{27}$ (f) $\frac{2}{-15}, \frac{1}{-12}$

3. Verify the following –

(a) $\frac{9}{-14} + \frac{17}{-21} = \frac{17}{-21} + \frac{9}{-14}$ (b) $-6 + \frac{-11}{-12} + \frac{-11}{-12} + (-6)$

(c) $-1 + \left(\frac{-2}{3} + \frac{-3}{4}\right) = \left(-1 + \frac{-2}{3}\right) + \frac{-3}{4}$ (d) $\left(\frac{-7}{11} + \frac{2}{-5}\right) + \frac{-13}{22} = \frac{-7}{11} + \left(\frac{2}{-5} + \frac{-13}{22}\right)$

(e) $-20 + \left(\frac{3}{-5} + \frac{-7}{-10}\right) = \left(-20 + \frac{3}{-5}\right) + \frac{-7}{-10}$

4. Find additive inverse of each of the following rational numbers –

(a) $\frac{21}{-40}$ (b) $\frac{-21}{30}$ (c) $\frac{-15}{-11}$ (d) 0 (e) $\frac{8}{-29}$

(f) $\frac{-17}{9}$ (g) $\frac{-23}{1}$ (h) $\frac{17}{9}$ (i) $\frac{2}{3}$

5. Subtract the following :

(a) $\frac{-4}{5}$ from $\frac{9}{8}$ (b) $\frac{-1}{16}$ from $\frac{-3}{8}$ (c) $\frac{3}{-4}$ from $\frac{4}{5}$

(d) $\frac{-4}{15}$ from $\frac{3}{10}$ (e) $\frac{4}{9}$ from $\frac{-1}{6}$ (f) $\frac{1}{5}$ from $\frac{3}{5}$

6. Find the sum using rearrangement property –

(a) $\frac{-11}{5} + \frac{-2}{3} + \frac{3}{5} + \frac{4}{3}$ (b) $\frac{3}{8} + \frac{-11}{6} + \frac{-1}{4} + \frac{-8}{3}$

(c) $\frac{-13}{20} + \frac{11}{14} + \frac{-5}{7} + \frac{7}{10}$ (d) $\frac{-6}{7} + \frac{-5}{6} + \frac{-4}{9} + \frac{-15}{7}$

7. What rational number should be subtracted from $\frac{-2}{3}$ to get $\frac{-1}{6}$?

8. What rational number should be added to –1 to get $\frac{5}{7}$?

9. Fill in the blanks:

(a) $\left(\frac{-12}{5}\right) + ... = \left(\frac{-3}{17}\right) + \left(\frac{-12}{5}\right)$ (b) $(-9) + \left(\frac{-31}{8}\right) = ... + (-9)$

(c) $(..........) + \frac{3}{7} + \frac{-13}{4} = \left(\frac{-8}{13} + \frac{3}{7}\right) + \left(\frac{-13}{4}\right)$ (d) $-12 + \left[\frac{7}{12} + \left(\frac{-9}{11}\right)\right] = \left[(-12 + \frac{-2}{3}\right] + ...$

(e) $\frac{19}{-5} + \left[\left(\frac{-3}{11}\right) + \left(\frac{-7}{8}\right)\right] = \left(\frac{19}{-5} + ...\right) + \frac{-7}{8}$ (f) $\frac{-16}{7} + ... = ... + \left(\frac{-16}{7}\right) = \frac{-16}{7}$

10. Verify that $-(-a) = a$, when $a =$ (a) $\frac{7}{6}$ (b) $\frac{-8}{9}$

11. **Verify that $-(a+b) = (-a) + (-b)$, when –**

(a) $a = \frac{3}{4}$, $b = \frac{3}{4}$ (b) $a = \frac{-3}{4}$, $b = \frac{-6}{7}$,

12. What should be subtracted from the sum of $\left(\frac{2}{5}+\frac{3}{4}+\frac{1}{3}\right)$ to get $\frac{1}{2}$?

13. **Simplify –**

(a) $\frac{13}{6}+\left(\frac{-2}{3}\right)+\left(\frac{-5}{6}\right)+\frac{11}{9}+\frac{1}{3}+\left(\frac{-2}{9}\right)$ (b) $\frac{-1}{3}+\frac{10}{7}+\left(\frac{-1}{6}\right)+\left(\frac{-5}{7}\right)+\frac{1}{12}+\frac{3}{4}$

14. **Write true or false –**

(a) If $|a| = 0$, then $a = 0$. ....

(b) If $|a| = |b|$, then $a = b$. ....

(c) If $\frac{a}{b} < \frac{c}{d}$, then $\frac{|a|}{|b|} < \frac{|c|}{|d|}$. ....

15. **Fill in the blank space with one of the following symbols. >, < or = :**

(a) If $\frac{-5}{7} < \frac{6}{13}$, then $\frac{|-5|}{|7|}$ .... $\frac{|6|}{|13|}$

(b) If $\frac{-5}{5} < \frac{-5}{6}$, then $\frac{|-5|}{|5|}$ .... $\frac{|-5|}{|6|}$

(c) If $\frac{-7}{8} < \frac{21}{24}$, then $\frac{|-7|}{|8|}$ .... $\frac{|21|}{|24|}$

(d) If $\frac{-9}{-10} > \frac{8}{9}$, then $\frac{|-9|}{|-10|}$ .... $\frac{|8|}{|9|}$

(e) If $\frac{-1}{2}+\frac{-3}{2}=\frac{-4}{2}$, then $\frac{|-1|}{|2|}+\frac{|-3|}{|2|}$ .... $\frac{|-4|}{|2|}$

Multiplication of Rational Numbers

Product of Rational Numbers–

$\frac{a}{b}\times\frac{c}{d}$ = a rational number if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers : (closure property).

Example 1 : $\frac{-2}{3}\times\frac{5}{7}=\frac{-10}{21}$ which is a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ rational numbers then, $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$ Commutative property of multiplication.

According to this property, the rational numbers can be multiplied in any order.

Example 2 : $\left(\frac{5}{7}\times\frac{3}{4}\right)=\left(\frac{3}{4}\times\frac{5}{7}\right)=\frac{15}{28}$

$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ – Associative property of multiplication.

This property states that while multiplying three or more rational numbers they can be grouped in any order.

Example 3 : $\left[\left(\frac{-5}{2}\right)\times\left(\frac{-7}{4}\right)\right]\times\frac{1}{3}=\frac{-5}{2}\times\left[\left(\frac{-7}{4}\right)\times\left(\frac{1}{3}\right)\right]=\frac{35}{24}$

$\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$ – Multiplicative property by 1.

When a rational number is multiplied by 1 the product is the rational number itself.

Example 4 : $\frac{3}{7}\times 1=1\times\frac{3}{7}=\frac{3}{7}$

$\frac{a}{b}\times 0=0\times\frac{a}{b}=0$ – Multiplicative property of O.

This law states that when a rational number is multiplied by 0, the product is 0 .

Example 5 : $\frac{9}{11}\times 0=0\times\frac{9}{11}=0$

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right)$ – Distributive property of multiplication over addition.

Example 6 : $\frac{-3}{4}\times\left(\frac{2}{3}+\frac{-5}{6}\right)=\left(\frac{-3}{4}\times\frac{2}{3}\right)+\left(\frac{-3}{4}\times\frac{-5}{6}\right)=\frac{3}{24}=\frac{1}{8}$

$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)-\left(\frac{a}{b}\times\frac{e}{f}\right)$ – Distributive property of multiplication over subtraction.

Example 7 : $\frac{1}{2}\times\left(\frac{5}{9}-\frac{2}{9}\right)=\left(\frac{1}{2}\times\frac{5}{9}\right)-\left(\frac{1}{2}\times\frac{2}{9}\right)$

$=\frac{5}{18}-\frac{2}{18}=\frac{5-2}{18}=\frac{3}{18}=\frac{1}{6}$

$\frac{a}{b}\times\frac{b}{a}=1$ – (existence of multiplicative inverse or reciprocal).

The multiplicative inverse of rational number $\frac{a}{b}$ is $\frac{b}{a}$.

Example 8 : What is then multiplication inverse of $\frac{3}{5}$?

$\frac{\cancel{3}^{1}}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{\cancel{3}_{1}}=\frac{1}{1}\times\frac{1}{1}=1$

Example : Verify the following :

(i) $\left(\frac{8}{15}\times\frac{-3}{16}\right)=\left(\frac{-3}{16}\times\frac{8}{15}\right)$

(ii) $\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right)=\left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$

(iii) $\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right)=\left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$

Solution : (i) $\left(\frac{8}{15}\times\frac{-3}{16}\right)=\left(\frac{-3}{16}\times\frac{8}{15}\right)$

$$= \left(\frac{1\times-1}{5\times2}\right) \quad \left(\frac{1\times-1}{5\times2}\right)$$

$$= \frac{-1}{10} = \frac{-1}{10}$$

LHS = RHS

$$= \left(\frac{8}{15}\times\frac{-3}{16}\right) = \left(\frac{-3}{16}\times\frac{8}{15}\right)$$

(ii) $$\frac{2}{3}\times\left(\frac{6}{7}\times\frac{-14}{15}\right) = \left(\frac{2}{3}\times\frac{6}{7}\right)\times\frac{-14}{15}$$

$$= \frac{2}{3}\times\frac{-12}{15} = \frac{4}{7}\times\frac{-14}{15}$$

$$= \frac{-8}{15} = \frac{-8}{15}$$

LHS = RHS Hence verified.

(iii) $$\frac{5}{6}\times\left(\frac{-4}{5}+\frac{-7}{10}\right) = \left(\frac{5}{6}\times\frac{-4}{5}\right)+\left(\frac{5}{6}\times\frac{-7}{10}\right)$$

$$= \frac{5}{6}\times\left(\frac{-8(-7)}{10}\right) = \frac{-20}{30}+\frac{-35}{60}$$

$$= \frac{5}{6}\times\frac{-15}{10} = \frac{-40+(-35)}{60}$$

$$= \frac{-5}{4} = \frac{-75}{60}$$

$$= \frac{-5}{4} = \frac{-5}{4}$$

LHS = RHS. Hence verified.

Exercise 1.3

1. Verify the following and state the laws used.

(a) $\frac{-17}{8}\times\frac{-11}{7}=\frac{-11}{7}\times\frac{-17}{8}$

(b) $\left(\frac{-2}{5}\times\frac{7}{11}\right)\times\frac{-11}{5}=\frac{-2}{5}\times\left(\frac{7}{11}\times\frac{-11}{5}\right)$

(c) $\frac{-1}{2}\times\left(\frac{-5}{6}\times\frac{7}{8}\right)=\left(\frac{-1}{2}\times\frac{-5}{6}\right)\times\frac{7}{8}$

(d) $\frac{-16}{9}\times1=1\times\frac{-16}{9}=\frac{-16}{9}$

(e) $\frac{-11}{19}\times\frac{19}{-11}=\frac{19}{-11}\times\frac{-11}{19}=1$

(f) $\frac{7}{5}\times0=0$

2. Answer then following question in short –

(a) What is the product of a rational number and its reciprocal?

(b) Does '0' have a reciprocal?

(c) What are the reciprocal of 1 and –1 respectively?

(d) Can zero be a reciprocal y/x , where x=0?

(e) What is the multiplicative reciprocal of a positive rational number ‘a’?

(f) What is the multiplicative reciprocal of a negative rational number ‘–a’?

3. **Find the products –**

(a) $\frac{5}{-18} \times \frac{-9}{20}$ (b) $\frac{-13}{15} \times \frac{-25}{26}$ (c) $\frac{16}{-21} \times \frac{14}{5}$ (d) $\frac{-7}{6} \times 24$

(e) $\frac{7}{24} \times (-48)$ (f) $\frac{-13}{5} \times (-10)$ (g) $\frac{3}{-5} \times \frac{-7}{8}$ (h) $\frac{-9}{2} \times \frac{5}{4}$

4. **Fill in the blanks –**

(a) $\frac{-21}{17} \times \frac{18}{35} = \frac{18}{35} \times$............. (b) $28 \times \frac{-7}{19} = \frac{-7}{19} \times$.............

(c) $\left(\frac{15}{7} \times \frac{-21}{10}\right) \times \frac{-5}{6} =$............... $\times \left(\frac{-21}{10} \times \frac{-5}{6}\right)$ (d) $\frac{-12}{7} \times \left(\frac{4}{15} \times \frac{25}{-19}\right) = \left(\frac{-12}{7} \times \frac{4}{15}\right) \times$.............

5. **Verify the following –**

(a) $\left(\frac{3}{4} \times \frac{1}{2}\right) \times \frac{3}{7} = \frac{3}{4} \times \left(\frac{1}{2} \times \frac{3}{7}\right)$ (b) $\left(\frac{-5}{6} \times \frac{-2}{5}\right) \times \frac{3}{7} = \frac{-5}{6} \times \left(\frac{-2}{5} \times \frac{3}{7}\right)$

(c) $\frac{7}{8} \times \left(\frac{2}{4} + \frac{4}{5}\right) = \left(\frac{7}{8} \times \frac{2}{4}\right) + \left(\frac{7}{8} \times \frac{4}{5}\right)$ (d) $\frac{-3}{7} \times \left(\frac{7}{8} + \frac{-5}{12}\right) = \left(\frac{-3}{7} \times \frac{7}{8}\right) + \left(\frac{-3}{7} \times \frac{-5}{12}\right)$

6. **Simplify using the properties of multiplication over addition and multiplication over subtraction of rational numbers.**

(a) $\frac{-3}{8} \times \left(\frac{4}{7} + \frac{-11}{7}\right)$ (b) $\frac{-2}{5} \times \left(\frac{3}{8} - 25\right)$ (c) $\frac{7}{4} \times \left(\frac{5}{8} + \frac{1}{2}\right)$

7. **Let a, b and c be three rational numbers having the values $a = \frac{-1}{3}$, $b = \frac{-3}{5}$ $c = \frac{-4}{9}$. Verify the following using the given values of a, b and c.**

(a) $a \times b = b \times a$ (b) $a \times (b \times c) = (a \times b) \times c$

(c) $a \times (b + c) = a \times b + a \times c$ (d) $(a - b)^{-1} = a^{-1} - b^{-1}$ is false

(e) $(a \times b)^{-1} = a + b^{-1}$ is false (f) $|a^{-1}| = |a|^{-1}$

(g) $|b^{-1}| = |b|^{-1}$ (h) $|c^{-1}| = |c|^{-1}$

(**Hint:** power of –1 is a sign of reciprocal, | | is a sign for finding absolute value)

8. What are the properties of multiplication involved in the equation $7 \times \frac{1}{7} x = x$?

9. Name the properties involved in the following –

$$42 \times \frac{1}{3} = (14 \times 3) \times \frac{1}{3} = (3 \times \frac{1}{3}) \times 14 = 1 \times 14 = 14$$

10. Find x if x is a rational number and $x \times x = x$

11. What are the two rational numbers, which are reciprocals of themselves ?

12. What is the reciprocal of x if $x \neq 0$?

13. Simplify – $\left[\left(\frac{2}{9}\right)^{-1}\right]^{-1}$

Division of Rational Numbers

(a) $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$ (b) $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

(c) $\frac{a}{b} \div \frac{c}{d}$, $\frac{a}{b}$ =dividend $\frac{c}{d}$ = divisor, result quotient

Example 1: Divide $\frac{36}{16}$ by $\frac{9}{8}$

Solution : $\frac{36}{16} \div \frac{9}{8} = \frac{\cancel{36}^{\cancel{4}\,2}}{\cancel{16}_{1\,2}} \times \frac{\cancel{8}^{1}}{\cancel{9}_{1}} = 2$

Example 2 : Divide $\frac{8}{23}$ by 1.

Solution : $\frac{8}{23} \div 1 = \frac{8}{23} \times \frac{1}{1} = \frac{8}{23}$

Example 3 : Divide $\frac{5}{9}$ by 0.

Solution $\frac{5}{9} \div 0$, not defined.

Facts to Know

- Babylonians developed tales of reciprocals. To divide a by b, they wrote a÷b = a: (1/b) (:=ratio =x)

Exercise 1.4

1. Divide –

(a) -18 by $\frac{-36}{37}$ (b) $\frac{-24}{50}$ by $\frac{-4}{75}$ (c) $\frac{-3}{16}$ by $\frac{-15}{18}$

(d) $\frac{10}{33}$ by $\frac{-2}{11}$ (e) $\frac{7}{18}$ by $\frac{-14}{51}$ (f) $\frac{5}{12}$ by 15

2. State whether the following are true or false –

(a) $\frac{-7}{24} \div \frac{3}{-16} = \frac{3}{-16} \div \frac{-7}{24}$ (b) $\frac{-4}{3} \div \frac{-8}{9} = \frac{-8}{9} \div \frac{-4}{3}$

(c) $-12 \div \frac{3}{4} = \frac{3}{4} \div -12$ (d) $\frac{-22}{7} \div \left(\frac{9}{14} - \frac{5}{21}\right) = \left(\frac{-22}{7} \div \frac{9}{14}\right) - \left(\frac{-22}{7} \div \frac{5}{21}\right)$

(e) $\left(\frac{9}{5} + \frac{4}{25}\right) \div \left(\frac{-5}{7}\right) = \frac{9}{5} \div \left(\frac{-5}{7}\right) + \frac{4}{25} \div \left(\frac{-5}{7}\right)$ (f) $\left(\frac{9}{20} - \frac{17}{40}\right) \div \frac{10}{3} = \left(\frac{9}{20} \div \frac{10}{3}\right) - \left(\frac{17}{40} \div \frac{10}{3}\right)$

3. Fill in the blanks –

(a) $\frac{-2}{9} \div \frac{-2}{9}$ =........... (b) $\frac{-4}{15} \div (-1)$ = ... (c) $\frac{12}{13} \div$ .... = – 1

(d) $\frac{6}{7} \div$ .... = $\frac{6}{7}$ (e) .... $\div 1 = \frac{-9}{17}$ (f) $\frac{-11}{25} \div$ .... =1

4. **Simplify –**

(a) $\frac{4}{1} \div \frac{-5}{12}$ (b) $-9 \div \frac{-7}{18}$ (c) $\frac{-12}{7} \div (-18)$

(d) $\frac{-1}{10} \div \frac{-8}{5}$ (e) $\frac{-16}{35} \div \frac{15}{14}$ (f) $\frac{-65}{14} \div \frac{13}{7}$

5. The product of two numbers is 6. If one number is 12, find the other number.

6. The product of two numbers is $\frac{-20}{9}$. If one number is $\frac{-4}{3}$, find the other number.

7. By what number should we multiply $\frac{-20}{63}$ to get $\frac{-5}{7}$?

8. By what number should $\frac{-8}{39}$ be multiplied to obtain $\frac{1}{26}$?

9. Divide the sum of $\frac{-12}{7}$ and $\frac{13}{5}$ by the product of $\frac{1}{-2}$ and $\frac{-31}{7}$.

10. Divide the sum of $\frac{8}{3}$ and $\frac{65}{12}$ by their difference.

11. **Write true or false –**

(a) We can divide 11 by 0. ....

(b) Rational numbers are always associative under division. ....

(c) Rational numbers are always commutative under division. ....

(d) Rational numbers are closed under division. ....

Exercise 1.5

1. Two pieces of lengths $4\frac{3}{5}$ and $2\frac{3}{10}$ have been cut off from a rope of 11*m*. Find the length of the remaining rope.

2. A container of sugar weighs $40\frac{1}{6}$ *kg*. If the weight of the container is $13\frac{3}{4}$ *kg*. Find the weight of sugar in it.

3. Find the cost of 35 *kg* of oranges if one kg of orange costs Rs. $46\frac{3}{4}$.

4. Find the area of a rectangular park which is $30\frac{3}{5}$ *m* long and $20\frac{2}{3}$ *m* wide.

5. A rope has been cut into 26 pieces. The total length of the rope is $71\frac{1}{2}$ *m*. Find the length of one piece of rope.

6. A rectangular room is $5\frac{7}{10}$ *m*. wide. Its area is $68\frac{2}{5}$ m^2. Find the length of the room.

7. The product of two fractions is $7\frac{3}{5}$. If one fraction is $4\frac{3}{7}$. Find the other fraction.

8. In a factory $\frac{5}{8}$ of the workers are women. There are 240 men. Find the number of people working in the factory.

9. How much distance will a bus cover in $7\frac{1}{2}$ hours if it is moving at a speed of $40\frac{2}{5}$ *km/hr* ?

10. Mr. Kohli sets out for his office with ₹ 80. He spend ₹ $5\frac{1}{2}$ as bus fare. ₹ $13\frac{3}{5}$ on snacks and $4\frac{2}{5}$ on repair of his shoes. How much money was left with him when he returned back home ?

11. Japneet gave 9 of grapes to the guests, 40 grapes were left in the bowl. How many grapes did the bowl contain ?

12. On the Independence day celebrations $\frac{2}{7}$ of the audience were seated. While 15000 were standing. Find the total number of the audience.

13. If jane earns ₹ 16000 per month. She spends $\frac{1}{4}$ of her salary on food, $\frac{1}{10}$ of her salary is sent to her parents, she spends $\frac{1}{4}$ of her salary on conveyance. How much is she able to save each month?

14. Aman gets ₹ 300 as pocket money each month he spends $\frac{1}{3}$ of his pocket money to eat fast foods. ¼ of the money is spent on chocolates. How much money is left with him.

Summary of Facts Discussed

1. A number of the form of $\frac{p}{q}$, where p and q are integers and q ≠ o is called a rational number.

2. Properties of rational numbers can be discussed on the four basic operations of mathematics. They are :
   (i) Addition (+) (ii) Subtraction (–)
   (iii) Multiplication (×) (iv) Division (÷)

3. The absolute value of a rational number is equal to its numerical value, which symbolically expressed as $\frac{a}{b}$ if a and b are integers.

4. **Closure properties of rational numbers :**
   The rational properties are closed under all the basic properties of operations. That is if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers then.
   a. $\frac{a}{b}+\frac{c}{d}$ is a rational number.
   b. $\frac{a}{b}-\frac{c}{d}$ is a rational number.
   c. $\frac{a}{b}\times\frac{c}{d}$ is a rational number.
   d. $\frac{a}{b}\div\frac{c}{d}$ is a rational number if $(\frac{c}{d} \neq 0)$.

5. **Commutative properties :**
   a. $\left(\frac{a}{b}+\frac{c}{d}\right)=\left(\frac{c}{d}+\frac{a}{b}\right)$ Commutative law of addition.
   b. $\left(\frac{a}{b}\times\frac{c}{d}\right)=\left(\frac{c}{d}\times\frac{a}{b}\right)$ Commutative law of multiplication.
   c $\left(\frac{a}{b}-\frac{c}{d}\right)\neq\left(\frac{c}{d}-\frac{a}{b}\right)$
   d. $\left(\frac{a}{b}\div\frac{c}{d}\right)\neq\left(\frac{c}{d}\div\frac{a}{b}\right)$

   Under operations of subtraction and division the rational numbers are not commutative.

6. **Associative properties :**
   Associative law states that rational numbers can be grouped in the desired way under the operations of multiplications and additions.

a. $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f}=\frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$ – Associative law of addition

b. $\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}=\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$ – Associative law of multiplication

c. $\left(\frac{a}{b}-\frac{c}{d}\right)-\frac{e}{f}=\frac{a}{b}-\left(\frac{c}{d}-\frac{e}{f}\right)$

d. $\left(\frac{a}{b}\div\frac{c}{d}\right)\div\frac{e}{f}=\frac{a}{b}\div\left(\frac{c}{d}\div\frac{e}{f}\right)$

7. Distributive Properties :

$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}+\frac{a}{b}\times\frac{e}{f}$ – Distributive property of multiplication over addition.

$\frac{a}{b}\times\left(\frac{c}{d}-\frac{e}{f}\right)=\frac{a}{b}\times\frac{c}{d}-\frac{a}{b}\times\frac{e}{f}$ – Distributive property of multiplication over subtraction.

8. Identity properties :

a. $\frac{a}{b}+0=0+\frac{a}{b}=\frac{a}{b}$ – zero additive identity.

b. $\frac{a}{b}-0=\frac{a}{b}$ – zero subtractive identity.

c. $\frac{a}{b}\times 0=0\times\frac{a}{b}=0$ – zero multiplicative identity.

d. $\frac{a}{b}\times 1=1\times\frac{a}{b}=\frac{a}{b}$ – multiplication identity.

e. $\frac{a}{b}\div 0$ – not defined

9. Inverse Identities :

a. $\frac{a}{b}+\frac{-a}{b}=\left(\frac{a}{b}\right)-\frac{a}{b}=0$ – Additive inverse.

b. $\frac{a}{b}\times\frac{a}{b}=1$ or $\frac{a}{b}\times\left(\frac{b}{a}\right)^{-1}\times\frac{a}{b}=1\times\frac{1}{\frac{a}{b}}$ – Multiplicative inverse or reciprocal of $\frac{a}{b}$.

c. $\left[\left(\frac{a}{b}\right)^{-1}\right]^{-1}=\frac{a}{b}$ – Reciprocal of the reciprocal of any number is the number itself.

10. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers then, $\frac{1}{2}\left(\frac{a}{b}+\frac{c}{d}\right)$ is a rational number lying between $\frac{a}{b}$ and $\frac{c}{d}$.

11. There are infinite numbers of rational numbers between $\frac{a}{b}$ and $\frac{c}{d}$.

Points to Remember :

- The integer p in the rational number $\frac{p}{q}$ is called its numerator and q is called its denominator.
- A rational number is said to be positive if its numerator and denominator are either both positive integers or both negative integers.
- Rational numbers are closed under addition, subtraction, multiplication, and division.
- Rational numbers are commutative and associative under addition and multiplication.
- Zero is the additive identity and 1 is the multiplicative identity for rational numbers.
- For a given rational number $\left(\frac{-p}{q}\right)$, there exists on additive inverse $\frac{p}{q}+\left(\frac{-p}{q}\right)=0$ such that $\frac{p}{q}+\left(\frac{-p}{q}\right)=0$
- For a given rational number $\frac{p}{q}\times\frac{q}{p}=1$, such that
- In rational numbers, multiplication distributes over addition and subtraction.
- There exist infinite rational numbers between two given numbers.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following is the additive inverse of $\frac{5}{9}$?

(i) $\frac{-5}{9}$ (ii) $\frac{9}{5}$ (iii) $-\frac{9}{5}$ (iv) $\frac{1}{8}$

(b) The sum of a rational number and its additive inverse is always –

(i) 1 (ii) 0 (iii) greater than 1 (iv) less than 1

(c) A rational number divided by zero is –

(i) 0 (ii) 1 (iii) not defined (iv) None of these

(d) The difference of $\frac{2}{3}-\frac{1}{7}$ is equal to –

(i) $\frac{-11}{21}$ (ii) $\frac{-21}{11}$ (iii) $\frac{11}{21}$ (iv) $\frac{1}{7}$

(e) The product of $\frac{3}{19}$ and its multiplication inverse is –

(i) 2 (ii) $\frac{1}{19}$ (iii) $\frac{19}{3}$ (iv) 1

(f) The sum of $\frac{5}{9}+\frac{1}{5}$ is equal to –

(i) $\frac{6}{14}$ (ii) $\frac{34}{45}$ (iii) $\frac{14}{6}$ (iv) $\frac{45}{34}$

(g) Which one is the commutative properties of rational number?

(i) $\frac{p}{q}+\frac{q}{p}=\frac{p}{q}+\frac{p}{q}$ ☐ (ii) $\frac{p}{q}+0=\frac{p}{q}-\frac{p}{q}$ ☐

(iii) $\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q}$ ☐ (iv) $\frac{p}{q}+-1=1-\frac{p}{q}$ ☐

(h) Multiplicative inverse of $\frac{-15}{29}$ is –

(i) $\frac{15}{19}$ ☐ (ii) $\frac{29}{15}$ ☐ (iii) $\frac{-15}{-19}$ ☐ (iv) $\frac{29}{-15}$ ☐

2. Name the property of a addition used in each of the following :

(a) $\left(\frac{-6}{7}\right)+\left(\frac{6}{7}\right)=0=\left(\frac{6}{7}\right)+\left(\frac{-6}{7}\right)$ (b) $\left(\frac{2}{9}+\frac{3}{5}\right)$ is a rational number

(c) $\frac{22}{39}+\left(\frac{-22}{39}\right)=0$ (d) $\frac{5}{7}+\left(\frac{-9}{19}\right)=\left(\frac{-9}{19}\right)+\frac{5}{7}$

(e) $\frac{1}{6}+\left(\frac{15}{39}+\frac{2}{11}\right)=\left(\frac{1}{6}+\frac{15}{39}\right)+\frac{2}{11}$ (f) $\frac{1}{18}+0=0+\frac{1}{18}=\frac{1}{18}$

3. Write the additive inverse of each of the following :

(a) $\frac{3}{7}$ (b) $\frac{-5}{11}$ (c) $\frac{15}{-7}$ (d) $\frac{-7}{-3}$

(e) $2\frac{1}{5}$ (f) $\frac{18}{-21}$ (g) $\frac{5}{19}$ (h) $\frac{-18}{-23}$

4. Write the multiplication inverse of each of the following :

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) -7 (d) $-3\times\frac{-2}{7}$

(e) $\frac{-99}{101}$ (f) $2\frac{1}{9}$ (g) $\frac{-1}{91}$ (h) $\frac{20}{-27}$

5. Name the property of multiplication used in each of the following :

(a) $\frac{2}{9}\times\left(\frac{1}{7}+\frac{2}{5}\right)=\frac{2}{9}\times\frac{1}{7}+\frac{2}{9}\times\frac{2}{5}$ (b) $\frac{17}{21}\times\left(\frac{23}{45}\times\frac{18}{51}\right)=\left(\frac{17}{21}\times\frac{23}{45}\right)\times\frac{18}{51}$

(c) $\frac{3}{4}\times\left(\frac{2}{7}-\frac{3}{5}\right)=\frac{3}{4}\times\frac{2}{7}-\frac{3}{4}\times\frac{3}{5}$ (d) $\frac{78}{103}\times1=1\times\frac{78}{103}=\frac{78}{103}$

(e) $\left(\frac{-41}{67}\right)\times\frac{8}{21}=\frac{8}{21}\times\left(\frac{-41}{67}\right)$ (f) $\frac{-15}{6}\times\frac{6}{-15}=1$

6. Simplify the following :

(a) $\left(\frac{-8}{7}\right)\times\frac{2}{5}\times\frac{7}{15}\times\frac{1}{32}$ (b) $\frac{3}{5}\times\frac{-2}{7}+\frac{4}{35}-\frac{3}{10}\times\frac{2}{7}$

7. Represent the following rational numbers on the number line.

(a) -3 (b) $\frac{-3}{5}$ (c) $\frac{16}{11}$ (d) $\frac{5}{9}$ (e) $\frac{11}{15}$

HOTS

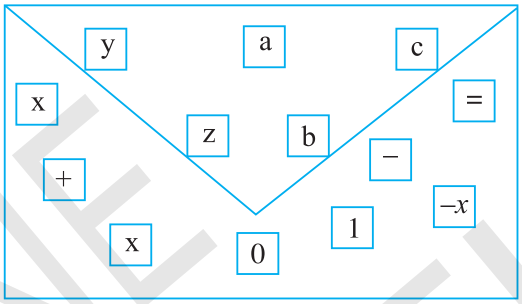
1. Nine times the reciprocal of a rational number equals 6 times the reciprocal of 17. Find the rational number.
2. Which rational numbers have absolute value less than 6 ?

Lab Activity

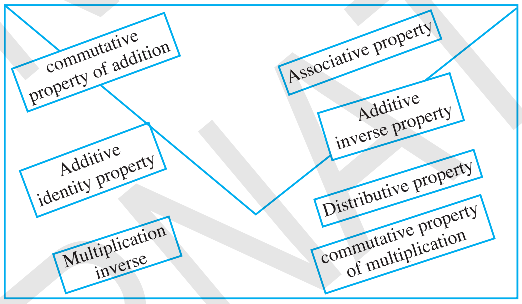
Objective : To understand the properties of rational numbers through activity.

Materials Required : Two envelopes : one envelope containing cards on which rational numbers and symbols are written and the other envelop containing strips on which properties of rational numbers are written.

Envelope 1



Envelope 2



Procedure : This game is played between two students. (Student A and Student B)

Step 1 : Student A is asked to take out a strip from envelope 2 randomly.

Step 2 : Student B is asked to choose number cards and symbol cards from Envelope 1 and demonstrate the property shown on the strip.

Step 3 : Each correct answer gets 2 marks and each wrong answer gets 1 negative mark.

Step 4 : The student who gets more marks will be judged the winner.

For example : Student A chooses the strip commutative property of addition.

Student B demonstrates the property :

x	+	y	=	y	+	x

2

Exponents and Powers

In earlier classes we have studied about how integers can be expressed in the form of power.

Example : $3^4 = 3\times3\times3\times3$ $\quad -2^4 \quad -2\times-2\times-2\times-2 = 2^4$

$-5^3 = -5\times-5\times-5 = -5^3$ $\quad -2^3 \quad -2\times-2\times-2 = -2^3$

In the number 3^4, 3 is called the **BASE** and the number 4, is called the **POWER**, or **EXPONENT** or **INDEX** of the number. The number 3^4 is read as " three raised to the power four".

For all positive integers *a* and *n* we have

$$(-a)^n = \begin{cases} a^n, \text{When n is even} \\ -a^n, \text{when } n \text{ is odd} \end{cases}$$

The system of writing numbers in this form is called **POWER NOTATION.**

Let *x* be a number then $x^m = x\times x\times x\times x$.....m times. In this class we will extend the system of power notation to rational numbers.

The base as well as the exponents can be positive or negative.

Positive Integral Exponent of a Rational Number

Let $\frac{p}{q}$ be any rational number and n be a positive integer, then.

$$\left(\frac{p}{q}\right)^n = \frac{p}{q}\times\frac{p}{q}\times\frac{p}{q}\text{.... } n \text{ times}$$

Thus $\left(\frac{p}{q}\right)^n = \frac{p^n}{q^n}$ for every positive integer '*n*'.

Example 1 : Simplify and evaluate the following.

(a) $\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{-3}{4}\right)^4$ (c) $\left(\frac{-2}{3}\right)^5$

Solution :

(a) $\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3} = \frac{3\times3\times3}{5\times5\times5} = \frac{27}{125}$

(b) $\left(\frac{-3}{4}\right)^4 = \frac{-3^4}{4^4} = \frac{-3\times-3\times-3\times-3}{4\times4\times4\times4} = \frac{81}{256}$

(c) $\left(\frac{-2}{3}\right)^5 = \frac{-2^5}{3^5} = \frac{-2\times-2\times-2\times-2\times-2}{3\times3\times3\times3\times3} = \frac{-32}{243}$

Negative Integral Exponent of a Rational Number

$$\left(\frac{p}{q}\right)^{-n} = \left(\frac{q}{p}\right)^n = \frac{q^n}{p^n} = \frac{q\times q\times q\text{....}n\text{ times}}{p\times p\times p\text{....}n\text{ times}}$$

Example 2: Evaluate the following.

(a) $\left(\frac{2}{5}\right)^{-1}$ (b) $\left(\frac{3}{5}\right)^{-3}$ (c) 4^{-4} (d) $\left(\frac{-3}{5}\right)^3$

Solution : (a) $\left(\frac{2}{5}\right)^{-1} = \left(\frac{5}{2}\right)^{1} = \frac{5}{2}$

(b) $\left(\frac{3}{5}\right)^{-3} = \left(\frac{5}{3}\right)^{3} = \frac{5^3}{3^3} = \frac{125}{27}$

(c) $(4)^{-4} = \left(\frac{1}{4}\right)^{4} = \frac{1^4}{4^4} = \frac{1}{256}$

(d) $\left(\frac{-3}{5}\right)^{-3} = \left(\frac{5}{-3}\right)^{3} = \frac{5^3}{-3^3} = \frac{125}{-27}$

Example 3 : Evaluate —

(a) $\left(\frac{1}{2}\right)^{0}$ (b) $\left(\frac{-3}{5}\right)^{0}$ (c) $\left(\frac{3}{4}\right)^{0}$

Solution : (a) $\left(\frac{1}{2}\right)^{0} = 1$ (b) $\left(\frac{-3}{5}\right)^{0} = 1$ (c) $\left(\frac{3}{4}\right)^{0} = 1$

Expressing Rational Numbers in Exponential form to Standard form and Vice Versa

Rational numbers in exponential form can be converted to standard form and vice versa using laws of exponents.

Example 1: Express the following rational numbers in standard form.

(a) $\left(\frac{7}{9}\right)^{3}$ (b) $\left(\frac{-5}{11}\right)^{4}$ (c) $\left(\frac{69}{72}\right)^{2}$ (d) $\left(\frac{21}{-25}\right)^{3}$

Solution : (a) $\left(\frac{7}{9}\right)^{3} = \frac{7^3}{9^3} = \frac{343}{729}$ (b) $\left(\frac{-5}{11}\right)^{4} = \frac{-5^4}{11^4} = \frac{625}{14641}$

(c) $\left(\frac{69}{72}\right)^{2} = \frac{69^2}{72^2} = \frac{4761}{5184} = \frac{529}{576}$ (d) $\left(\frac{21}{-25}\right)^{3} = \frac{21^3}{-25^3} = \frac{9261}{-15625} = \frac{-9261}{15625}$

To bring a rational number in the standard form it should be reduced to the lowest term if its denominators are negative. It should be changed to negative. It should be changed to positive.

Example 2: Express the following rational numbers in exponential form.

(a) $\frac{-343}{729}$ (b) $\frac{-49}{64}$ (c) $\frac{81}{625}$ (d) $\frac{-32}{-243}$ (e) $\frac{16}{-49}$ (f) $\frac{8}{125}$

Solution : (a) $\frac{-343}{729} = \frac{-7^3}{9^3} = \left(\frac{-7}{9}\right)^{3}$ (d) $\frac{-32\times-1}{-243\times-1} = \frac{32}{243} = \frac{2^5}{3^5} = \left(\frac{2}{3}\right)^{5}$

(b) $\frac{-49}{64} = \left(\frac{-7^2}{8^2}\right) = \left(\frac{-7}{8}\right)^{2}$ (e) $\frac{16\times-1}{-49\times-1} = \frac{16}{49} = \frac{4^2}{7^2} = \left(\frac{4}{7}\right)^{2}$

(c) $\frac{81}{625} = \frac{3^4}{5^5} = \left(\frac{3}{5}\right)^{4}$ (f) $\frac{8}{125} = \frac{2^3}{5^3} = \left(\frac{2}{5}\right)^{3}$

$$a^{-n} = \frac{1}{a^n}$$

(i) $(5)^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$ (ii) $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{-3 \times -3} = \frac{1}{9}$

Example 1 : Evaluate each of the following.

(i) $5^2 \times 5^4$, (ii) $5^8 \div 5^3$ (iii) $(3^2)^3$ (iv) $\left(\frac{11}{12}\right)^3$ (v) $\left(\frac{3}{4}\right)^{-3}$

Solution : (i) $5^2 \times 5^4 = 5^{2+4} = 5^6 = 15625.$

(ii) $5^8 \div 5^3 = \frac{5}{5} = 5^{8-3} = 5^5 = 3125$

(iii) $(3^2)^3 = 3^{2 \times 3} = 3^6 = 729$

(iv) $\left(\frac{11}{12}\right)^3 = \frac{11^3}{12^3} = \frac{1331}{1728}$

(v) $\left(\frac{3}{4}\right)^{-3} = \frac{1}{\left(\frac{3}{4}\right)^{-3}} = \frac{1}{\frac{(3)^3}{(4)^3}} = \frac{1}{\frac{27}{64}} = \frac{64}{27}$

Example 2 : Using laws of exponent simplify.

Solution : $(3^{-1} \times 5^{-1})^{-1} \div 7^{-1}$

$= \left(\frac{1}{3} \times \frac{1}{5}\right)^{-1} \div \left(\frac{1}{7}\right)$

$= \left(\frac{1}{15}\right)^{-1} \times 7$

$= \frac{15}{1} \times \frac{7}{1} = 105$

Example 3 : If $a = 2$ and $b = 3$ then find the values of each of the following.

Solution : (i) $a^b + b^a = 2^2 + 3^3 = 4 + 27 = 31.$

(ii) $a^b + b^a = 2^3 + 3^2 = 8 + 9 = 17.$

(iii) $\left(\frac{1}{a} + \frac{1}{b}\right)^a = \left(\frac{1}{2} + \frac{1}{3}\right)^2 = \left(\frac{3+2}{3 \times 2}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{5^2}{6^2} = \frac{25}{36}$

Example 4 : Find the value of x if.

(i) $3^x = 81$ (ii) $2^{x-3} = 1$ (iii) $3^{3x-5} = \frac{1}{9^x}$

Solution : (i) $3^x = 81$

$3^x = 3^4$

$x = 4$

(ii) $2^{x-3} = 1$

$= 2^{x-3} = 2^0$

$= x - 3 = 0 = x = 3$

(iii) $3^{3x-5} = \frac{1}{9^x}$

$3^{3x-5} = \frac{1}{3^{2x}} = 3^{3x-5} = 3^{-2x}$

$3x - 5 = -2x = 3x + 2x = 5$

$5x = 5 = x = 1$

Example 5 : Using prime factorization, express 144×750 as the product of prime factors.

Solution : By prime factorization, we have

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	750
3	375
5	125
5	25
5	5
	1

$\therefore$ 144 = $2\times2\times2\times2\times3\times3 = 2^4\times3^2$

and, 750 = $2\times3\times5\times5\times5 = 2\times3\times5^3$

So, 144×750 = $2^4\times3^2\times2\times3\times5^3 = (2^5\times3^3\times5^3)$

Example 6 : Solve each of the following exponential equations.

(i) $7^x = 343$ (ii) $2^{x-3} = 1$

Solution : (i) $7^x = 343 \Rightarrow 7^x = 7^3 \Rightarrow x = 3$

(ii) $2^{x-3} = 1 \Rightarrow 2^{x-3} = 2^0$ $[\because 2^0 = 1]$ $\Rightarrow x-3 = 0 \Rightarrow x = 3.$

Exercise 2.1

1. Write the base and exponent in each of the following :

(a) 2^5 (b) $(-5)^4$ (c) $\frac{1}{3^4}$

2. Write each of the following in exponential form :

(a) $5\times5\times5\times5$ (b) $\frac{2}{7}\times\frac{2}{7}\times\frac{2}{7}\times\frac{2}{7}$ (c) $\frac{81}{625}$

(d) $(-2)\times(-2)\times(-2)\times(-2)\times(-2)$

3. Simplify and write the answer in the exponential form :

(a) $3^2\times\frac{1}{3^5}\times(3^3)^4$ (b) $(-5)^4\times(-5)^3\times(-5)^2$

4. Find the value of x:

(a) $5^{x-2} = 25$ (b) $2^{5x}\div2^x = 2^4$ (c) $(2^2)^x\div(2^3)^4 = 1$

5. (a) Express 729 as a power of 3. (b) Express 343 as a power of 7.

(c) Express –128 as a power of –2.

6. Find the value of *x*, if :

(a) $2^x + 2^x + 2^x = 192$ (b) $2^3 + 2^x = 2^4$ (c) $8^{255} = 32^x$

7. Evaluate the following :

(a) $3x^4 - (8x^2)^2 + 8(x^2)^2 + (3x^2)^2$ (b) $\left(\frac{12}{5}\right)^3\times\frac{5^6}{144}$ (c) $\frac{9^3\times27\times81^4}{3^{-2}\times3^4\times81^2}$

8. Express each of the following as a product of prime factors only :

(a) 108×192 (b) 363×132

9. Solve the following exponential equations:

(a) $6^{x-2} = 1$ (b) $(\sqrt{2})^x = 2^8$ (c) $3^{3x-5} = \frac{1}{9^x}$

Exercise 2.2

1. Express as a rational number in standard form :

(a) $\left(\frac{1}{3}\right)^5$ (b) $-\left(\frac{4}{27}\right)^2$ (c) $-\left(\frac{5}{11}\right)^4$ (d) $\left(\frac{7}{4}\right)^4$

2. Express in the exponential form :

(a) $\frac{9}{49}$ (b) $\frac{243}{1024}$ (c) $\left(\frac{16}{81}\right)$ (d) $\frac{-125}{729}$

3. Express as rational numbers:

(a) $\left(3^2-2^2\right)\div\left(\frac{1}{5}\right)^2$ (b) $\left[\left(\frac{1}{2}\right)^2-\left(\frac{1}{4}\right)^3\right]\times 2^3$ (c) $\left(\frac{1}{2}\right)^2\times 2^3\times\left(\frac{3}{4}\right)^2$

(d) $\left(\frac{3}{4}\right)^3\times\left(\frac{2}{3}\right)^2$ (e) $(-2)^5\div\left(\frac{-1}{3}\right)^3$ (f) $\left(\frac{1}{3}\right)^4\div\left(\frac{1}{9}\right)^6$

(g) $\left(\frac{-2}{3}\right)^4\times\left(\frac{-3}{4}\right)^3$ (h) $\left(\frac{3}{5}\right)^4\times\left(\frac{1}{3}\right)^3$

4. Find the reciprocal of the following rational numbers :

(a) $\left(\frac{3}{7}\right)^2$ (b) $\left(\frac{3}{4}\right)^5$ (c) $\left(\frac{-2}{3}\right)^4$ (d) $\left(\frac{-5}{9}\right)^3$

(e) $\left(\frac{1}{3}\right)^5$ (f) $\left(\frac{-7}{-4}\right)^4$ (g) $\left(\frac{5}{11}\right)^4$ (h) $\left(\frac{-4}{27}\right)^2$

5. Find the absolute values :

(a) $\left(\frac{5}{-3}\right)^4$ (b) $\left(\frac{-11}{13}\right)^2$ (c) $\left(\frac{2}{7}\right)^5$ (d) $\left(\frac{-1}{3}\right)^3$

6. State which rational number is greater $\frac{4}{3^2}$ or $\left(\frac{4}{3}\right)^2$

7. Find 12 rational numbers between $\frac{3^2}{4}$ and $\left(\frac{3}{4}\right)^2$

Laws of Exponents

Let $\frac{a}{b}$ be any rational and *m* and *n* be integers. Then we have the following identities.

(1) $\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n$

(2) $\left(\frac{a}{b}\right)^0=1$

(3) $\left(\frac{a}{b}\right)^m\times\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^{m+n}$

(4) $\left(\frac{a}{b}\right)^m\div\left(\frac{a}{b}\right)^n=\left(\frac{a}{b}\right)^{m-n}$ (if m>n)

(5) $\left(\frac{a}{b}\times\frac{c}{d}\right)^n=\left(\frac{a}{b}\right)^n\times\left(\frac{c}{d}\right)^n$

(6) $\left(\frac{a/b}{c/d}\right)^n=\frac{\left(\frac{a}{b}\right)^n}{\left(\frac{c}{d}\right)^n}$

(7) $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$

(8) $\left(\frac{a}{b}\times\frac{c}{d}\right)^{-n}=\frac{1}{\left(\frac{a}{b}\times\frac{c}{d}\right)^n}=\left(\frac{b}{a}\times\frac{d}{c}\right)^n$

Example : Evaluate and name the law of exponent used in evaluating the following rational numbers.

(a) $\left(\frac{2}{3}\right)^{-1}$ (b) $\left(\frac{3}{5}\right)^{-3}$ (c) $\left(\frac{7}{11}\right)^{0}$ (d) $\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}$

(e) $\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}$ (f) $\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}$ (g) $\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}$ (h) $\left(\frac{2/3}{5/7}\right)^{3}$ (i) $\left(\frac{7}{13}\times\frac{2}{11}\right)^{-2}$

Solution :

(a) $\left(\frac{2}{3}\right)^{-1}$ using the identity $\left(\frac{a}{b}\right)^{-1}=\frac{b}{a} \Rightarrow \left(\frac{2}{3}\right)^{-1}=\frac{3}{2}$

(b) $\left(\frac{3}{5}\right)^{-3}$ using the identity $\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^{m} \Rightarrow \left(\frac{3}{5}\right)^{-3}=\left(\frac{5}{3}\right)^{3}=\frac{125}{27}$

(c) $\left(\frac{7}{11}\right)^{0}$ using the identity $\left(\frac{a}{b}\right)^{0}=1 \Rightarrow \left(\frac{7}{11}\right)^{0}=1$

(d) $\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}$ using the identity $\left(\frac{a}{b}\right)^{m}\times\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m+n}$

$$\left(\frac{1}{2}\right)^{4}\times\left(\frac{1}{2}\right)^{3}=\left(\frac{1}{2}\right)^{4+3}=\left(\frac{1}{2}\right)^{7}=\frac{1}{128}$$

(e) $\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}$ using the identity $\left(\frac{a}{b}\right)^{m}\div\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m-n}$ when m > n.

$$\left(\frac{2}{5}\right)^{5}\div\left(\frac{2}{5}\right)^{2}=\left(\frac{2}{5}\right)^{5-2}=\left(\frac{2}{5}\right)^{3}=\frac{2^3}{5^3}=\frac{8}{125}$$

(f) $\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}$ using the identity $\left(\frac{a}{b}\right)^{m}\div\left(\frac{a}{b}\right)^{n}=\left(\frac{a}{b}\right)^{m-n}$ when m > n.

$$\left(\frac{2}{5}\right)^{-5}\div\left(\frac{2}{5}\right)^{-2}=\left(\frac{2}{5}\right)^{-5(-2)}=\left(\frac{2}{5}\right)^{-5+2}=\left(\frac{2}{5}\right)^{-3}$$

$=\frac{1}{\left(\frac{2}{5}\right)^{3}}$ using the identity $\left(\frac{a}{b}\right)^{-n}=\frac{1}{\left(\frac{a}{b}\right)^{n}}$ $=\left(\frac{5}{2}\right)^{3}=\frac{5^3}{2^3}=\frac{125}{8}$

(g) $\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}$ using the identity $\left(\frac{a}{b}\times\frac{c}{d}\right)^{m}=\left(\frac{a}{b}\right)^{m}\times\left(\frac{c}{d}\right)^{n}$

$$\left(\frac{3}{7}\times\frac{5}{11}\right)^{3}=\left(\frac{3}{7}\right)^{3}=\left(\frac{5}{11}\right)^{3}=\frac{3^3}{7^3}\times\frac{5^3}{11^3}=\frac{27}{343}\times\frac{125}{1331}=\frac{3375}{456533}$$

(h) $\left(\frac{2/3}{5/7}\right)^{3}$ using the identity $\left(\frac{a/b}{c/d}\right)^{m}=\frac{\left(\frac{a}{b}\right)^{m}}{\left(\frac{c}{d}\right)^{m}}$

$$\left(\frac{2/3}{5/7}\right)^{3}=\frac{\left(\frac{2}{3}\right)^{3}}{\left(\frac{5}{7}\right)^{3}}=\frac{\frac{2^3}{3^3}}{\frac{5^3}{7^3}}=\frac{\frac{8}{27}}{\frac{125}{343}}=\frac{8}{27}\times\frac{343}{125}=\frac{2744}{3375}$$

(i) $\left(\frac{7}{13}\times\frac{2}{11}\right)^{-2}$ using the identity $\left(\frac{a}{b}\times\frac{c}{d}\right)^{-n}=\frac{1}{\left(\frac{a}{b}\times\frac{c}{d}\right)^{n}}=\left(\frac{b}{a}\times\frac{d}{c}\right)^{n}$

$$=\left(\frac{7}{13}\times\frac{2}{11}\right)^{n2}=\frac{1}{\left(\frac{7}{13}\times\frac{2}{11}\right)^{2}}=\left(\frac{13}{7}\times\frac{11}{2}\right)^{2}=\left(\frac{143}{14}\right)^{2}=\frac{20449}{196}$$

Exercise 2.3

1. Write True or False for the following statements :

(a) $\left(\frac{-3}{60}\right)^{50}=\left|\left(\frac{3}{60}\right)^{50}\right|$

(b) The reciprocal of $\left(\frac{3}{7}\right)^{20}$ is $\left(\frac{7}{3}\right)^{20}$

(c) $(100)^{10}=\quad 1000^{10}$

(d) $\left[\left(\frac{1}{3}\right)^{3}\right]$ is reciprocal of 3^3

(e) $(110+110)^{25}=110^{25}+110^{25}$

2. Fill in the blanks :

(a) $(-5)^3\times(-5)^2=(-5)^{\square}$ ....

(b) $(13)^2\times(13)^5=13^{\square}$ ....

(c) $\left(\frac{1}{5}\right)^{7}\times\left(\frac{1}{5}\right)^{11}=\left(\frac{1}{5}\right)^{\square}$ ....

(d) $\left(\frac{2}{3}\right)^{8}\div\left(\frac{2}{3}\right)^{5}=\left(\frac{2}{3}\right)^{\square}$ ....

(e) $\left(\frac{-7}{13}\right)^{9}\div\left(\frac{-7}{13}\right)^{5}=\left(\frac{-7}{13}\right)^{\square}$ ....

(f) $2^{11}\div 2^{20}=\left(\frac{1}{2}\right)^{\square}$ ....

(g) $(-79)^3\div(-79)^8=\left(\frac{1}{-79}\right)^{\square}$ ....

Examples

Example 1 : Express the following in the form of rational numbers.

(a) 2^{-3} (b) $\left(\frac{1}{2}\right)^{-4}$ (c) $\left(\frac{3}{2}\right)^{-3}$ (d) $(-3)^{-2}$ (e) $\left(\frac{-5}{7}\right)^{-4}$

Solution :

(a) $2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

(b) $\left(\frac{1}{2}\right)^{-4}=\left(\frac{2}{1}\right)^{4}=\frac{16}{1}$

(c) $\left(\frac{3}{2}\right)^{-3}=\left(\frac{2}{3}\right)^{3}=\frac{2^3}{3^3}=\frac{8}{27}$

(d) $(-3)^{-2}=\frac{1}{-3^2}=\frac{1}{9}$

(e) $\left(\frac{-5}{7}\right)^{-4}=\left(\frac{7}{-5}\right)^{4}=\frac{7^4}{(-5)^4}=\frac{2401}{625}$

Example 2 : Evaluate the following.

(a) $\left(\frac{2}{5}\right)^{3}\times\left(\frac{2}{5}\right)^{2}$ (b) $\left(\frac{3}{7}\right)^{5}\times\left(\frac{3}{7}\right)^{-2}$ (c) $\left(\frac{2}{3}\right)^{-3}\times\left(\frac{2}{3}\right)^{-2}$

Solution :

(a) $\left(\frac{2}{5}\right)^{3}\times\left(\frac{2}{5}\right)^{2}=\left(\frac{2}{5}\right)^{3+2}=\left(\frac{2}{5}\right)^{5}=\frac{2^5}{5^5}=\frac{32}{3125}$

(b) $\left(\frac{3}{7}\right)^{5}\times\left(\frac{3}{7}\right)^{-2}=\left(\frac{3}{7}\right)^{5-2}=\left(\frac{3}{7}\right)^{3}=\frac{3^3}{7^3}=\frac{27}{343}$

(c) $\left(\frac{2}{3}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2} = \left(\frac{2}{3}\right)^{-3+(-2)} = \left(\frac{2}{3}\right)^{-3-2} = \left(\frac{2}{3}\right)^{-5} = \left(\frac{3}{2}\right)^{5} = \frac{3^5}{5^5} = \frac{243}{32}$

Example 3 : Simplify – $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{2}$

Solution : $\left(\frac{-2}{7}\right)^{-4} \times \left(\frac{-5}{7}\right)^{2} = \left(\frac{7}{-2}\right)^{4} \times \left(\frac{-5}{7}\right)^{2}$

$= \frac{-7^4}{2^4} \times \frac{-5^2}{7^2} = \frac{-7^4 \times -5^2}{2^4 \times 7^2} = \frac{-7^2 \times -5^2}{2^4} = \frac{49 \times 25}{16} = \frac{1225}{16}$

Example 4 : Simplify – $\left(\frac{-1}{3}\right)^{-5} \times \left(\frac{-1}{3}\right)^{-4}$

Solution : $\left(\frac{-1}{3}\right)^{-5} \times \left(\frac{-1}{3}\right)^{-4} = \left(\frac{3}{-1}\right)^{5} \times \left(\frac{3}{-1}\right)^{4} = (-3)^5 \times (-3)^4$

$= (-3)^{5+4} = (-3)^9 = -19683$

Example 5 : $(3^{-1} \times 7^{-1}) \div 4^{-1}$

Solution : $\left(\frac{1}{3} \times \frac{1}{7}\right)^{-1} \div 4^{-1} = \left(\frac{1}{21}\right)^{-1} \div \frac{1}{4}$

$21 \div \frac{1}{4} = \frac{\frac{21}{1}}{4} = \frac{21}{1} \div \frac{4}{1} = 21 \times 4 = 84$

Example 6 : $(2^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1}$

Solution : $(2^{-1} + 8^{-1}) \div \left(\frac{2}{3}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{8}\right) \div \frac{3}{2} = \left(\frac{4+1}{8}\right) \times \frac{2}{3} = \frac{5}{8} \times \frac{2}{3} = \frac{5}{4 \times 3} = \frac{5}{12}$

Example 7 : Simplify – $\left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2}$

Solution : $\left(\frac{1}{5}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{2}\right)^{-2}$

$= 5^2 + 3^2 + 2^2 = 25 + 9 + 4 = 38$

Example 8 : By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiply so that the product is $\left(\frac{-5}{4}\right)^{-1}$?

Solution : Let the number be *x*

$\left(\frac{1}{2}\right)^{-1} \times x = \left(\frac{-5}{4}\right)^{-1}$

$= 2 \times x = \frac{4}{-5}$

$= x = \frac{4}{-5} \div 2$

$= \frac{4}{-5} \times \frac{1}{2} = \frac{2}{-5} \times \frac{-1}{-1} = \frac{-2}{5}$

Example 9 : By what number should $\left(\frac{-2}{3}\right)^{-3}$ be divided so that the quotient is $\left(\frac{4}{27}\right)^{-2}$?

Solution : Let the number be *x*

$$\left(\frac{-2}{3}\right)^{-3} \div x = \left(\frac{4}{27}\right)^{-2}$$

$$= \frac{\left(\frac{3}{-2}\right)^{3}}{x} = \left(\frac{27}{4}\right)^{2}$$

$$= \frac{27}{-8} \times \frac{1}{x} = \frac{27 \times 27}{4 \times 4}$$

$$= \frac{1}{x} = \frac{27 \times 27}{4 \times 4} \times \frac{-8}{27}$$

$$= \frac{1}{x} = \frac{-27}{2}$$

$$= x \times -27 = 2$$

$$= x = \frac{2}{-27} \times \frac{-1}{-1} = \frac{-2}{27}$$

Example 10 : If $5^{2x+1} \div 25 = 125$, find the value of x.

Solution :

$$5^{2x+1} \div 25 = 125$$
$$= 5^{2x+1} \div 5^{2} = 5^{3}$$
$$= 5^{2x+1} = 5^{3} \times 5^{2}$$
$$= 5^{2x+1} = 5^{3+2}$$
$$= 2x+1 = 5$$
$$= 2x = 5-1 = 2x = 4$$
$$= x = \frac{4}{2} = x = 2$$

Exercise 2.4

1. Simplify the rational numbers and express with positive exponents :

(a) $\left(\frac{4}{25}\right)^{-3} \times \left(\frac{4}{25}\right)^{11} \times \left(\frac{4}{25}\right)^{-10}$ (b) $\left(\frac{-3}{5}\right)^{-6} \div \left(\frac{-3}{5}\right)^{2}$ (c) $\left(\frac{-8}{5}\right)^{-7} \div \left(\frac{-8}{5}\right)^{4}$

(d) $\left[\left(\frac{5}{7}\right)^{-3} \times \left(\frac{5}{7}\right)^{-7}\right] \times \left(\frac{5}{7}\right)^{-3}$ (e) $\left[\left(\frac{9}{11}\right)^{-2}\right]^{-4}$ (f) $\left[\left\{\left(\frac{-7}{11}\right)^{-3}\right\}^{-4}\right]^{-2}$

2. Evaluate : (a) $\left(\frac{1}{20}\right)^{-3} \times (-16)^{-3}$ (b) $\left(\frac{7}{44}\right)^{-4} \div \left(\frac{11}{7}\right)^{4}$ (c) $\left(\frac{-6}{5}\right)^{-2} \times \left(\frac{-3}{4}\right)^{-2}$

(d) $\left(\frac{-7}{8}\right)^{0} \times \left(\frac{3}{4}\right)^{-3} \times \left(\frac{2}{3}\right)^{-2}$ (e) $(2^{-1} \times 5^{-1})^{-1} \div 4^{-1}$ (f) $(4^{-1} + 8^{-1})^{-1} \div \left(\frac{2}{3}\right)^{-1}$

(g) $\left(\frac{-1}{4}\right)^{-3} \div \left(\frac{3}{8}\right)^{-2}$ (h) $(3^{-1} \div 4^{-1})^{2}$

3. Find reciprocal :

(a) $\left[\left(\frac{3}{7}\right)^{2}\right]^{5} \times \left(\frac{7}{3}\right)^{-12}$ (b) $\left(\frac{-5}{11}\right)^{-3} \div \left(\frac{-5}{11}\right)^{-4}$

4. Find the rational number which should be multiplied with $\left(\frac{-3}{2}\right)^{-3}$ so that the product is $\left(\frac{9}{8}\right)^{-2}$.

5. Find the rational number with which $(-6)^{-1}$ should be multiplied so that the product is 9^{-1}.

6. **Find the value of *x* If :**

(a) $7^{2x+1} \div 49 = 243$ (b) $3^{2x+1} \div 9 = 27$

7. If $x = \left(\frac{3}{4}\right)^{-2} \times \left(\frac{6}{9}\right)^{-2}$, find the value of x^{-3}.

8. **Show that:**

(a) $2^7 \times 2^{-5} = 2^{7+(-5)}$ (b) $(-27)^{-9} \times (-27)^5 = (-27)^{-9+5}$

(c) $\left(\frac{-2}{3}\right)^{-3} \times \left(\frac{-2}{3}\right)^{-4} = \left(\frac{-2}{3}\right)^{-3+(-4)}$ (d) $\left(\frac{8}{3} \times \frac{7}{11}\right)^2 = \left(\frac{7}{11}\right)^2 \times \left(\frac{8}{3}\right)^2$

9. **Find the value of *x* such that :**

(a) $\left(\frac{3}{4}\right)^{-5} \times \left(\frac{3}{4}\right)^{-3} = \left(\frac{3}{4}\right)^{x-2}$ (b) $\left(\frac{343}{8}\right)^{x} \times \left(\frac{343}{8}\right)^{5} = \left(\frac{7}{2}\right)^{18}$

Using Exponents for Scientific Notations

Sometimes we come across very large or very small numbers. Which when expressed in digits or numbers become very difficult to understand. Such numbers are written in the form of powers. In science we come across very large numbers such as —

(a) Distances of star, sun and moon etc.

(b) Ages of earth and universe.

(c) Speed of light and other rays.

As well as very small numbers such as —

(a) Size of atoms and molecules.

(b) Size of unicellular organisms.

(c) Size of blood cells and other cells.

Expressing these numbers in the form of powers is called Scientific Notation.

In scientific notation we simply shift the decimal after the extreme left non zero digit.

Example 1 : Change the number 9000000000000 to scientific notation.

Solution : 12 zeroes succeed the digit 9

it can be written as 9×10^{12}

Example 2 : Change the number 9365002.01 to scientific notation.

Solution : $\frac{9365002.01}{1 \times 10^6} \times 10^6$

$= 9.365002 \times 10^6$

$= 9.4 \times 10^6$ approximately.

Example 3 : Change 0.000000934 to scientific notation.

Solution : $\dfrac{0.000000934 \times 10^7}{10^7} = \dfrac{9.34}{10^7}$

$= 9.34 \times 10^{-7}$

Example 4 : Change $\dfrac{3}{1000000}$ into scientific notation.

Solution : $\dfrac{3}{1000000} = \dfrac{3}{10^6} = 3 \times 10^{-6}$

Short Cut Method

Example 1 : 9365002.01

Solution : 9.365002×10^6 or 9.4×10^6

Because the decimal has shifted six places towards the left.

Example 2 : $0.000000934 = 9.34 \times 10^{-7}$

Because the decimals has shifted seven places towards the right.

The digits on the left of the decimal are denoted by the letter k. The power of 10 is denoted by n. Therefore the scientific notations are numbers of the form of $k \times 10^n$.

That is, when we say that $n = 5$. It means that the power of the 10 is 5 or 10^5.

Example 3 : Change the number 936520003.03 to —

(a) $n = 5$ (b) $n = 7$ (c) $n = 8$

Solution : (a) $n = 5$

$= K \times 10^5$

Shift the point five places towards the left

9365.2000303×10^5 or 9365.2×10^5

(b) $n = 7$

$= K \times 10^7$

$= 93.652000303 \times 10^7$ or 93.65×10^7

(c) $n = 8$

$K \times 10^8$

$= 9.3652000303 \times 10^8$ or 9.4×10^8

Exercise 2.5

1. Express in scientific notation or in the form of $k \times 10^n$ with value of n given:

(a) 190000000, $n = 8$ (b) 12300000000, $n = 9$

(c) 0.0000000000000037, $n = -15$ (d) 0.0000000066, $n = -9$

2. Write the following numbers in the usual form:

(a) 9.5×10^7 (b) 9.8×10^9 (c) 6.5146939×10^7

(d) 3.8×10^{11} (e) 1.001×10^{10} (f) 6.5×10^5

3. **Express the number 71865000000 in the form of $k \times 10^n$, where:**

(a) $n = 10$ (b) $n = 9$ (c) $n = 7$ (d) $n = 6$ (e) $n = 8$

4. **Express the number 0.00003984 in the form of $K \times 10^n$ where :**

(a) $n = -7$ (b) $n = -6$ (c) $n = -5$

5. **Reexpress the following statements with their numbers in the form of scientific notation with k = 1.**

(Hint k = No of digits counted from left to right)

(a) The speed of light is approximately 300000 km/second in the vacuum.

(b) The speed of light is exactly 299792.5 km/second in the vacuum.

(c) The universe is approximately 8,000,000,000 years old.

(d) The earth is approximately 6,000,000,000 years old.

(e) The mean distance of sun from the earth is 150,000,000 km.

(f) The mass of the earth is 5980,000,000,000,000,000,000,000,kg.

(g) The unit angstrom (Å) is used to measure radii of atoms and molecules and wavelength.

(**Hint:** 1Å = 0.0000000001 m.)

(h) Each day in Delhi 1050000 kg of pollutants are released.

(i) The earth has 1,353,000,000 km^3 of sea water.

(j) The sea water contains 13,61,10,00000 kg of gold.

(k) The unit micron is used to measure microorganisms. 1 micron = $\frac{1}{1000,000}$ m.

Negative rational numbers as exponents

Example 5 : $(27)^{-2/3} = \left(\frac{1}{(27)^{2/3}}\right) = \left(\frac{1}{\left[(27)^2\right]^{1/3}}\right) = \left(\frac{1}{\sqrt[3]{27 \times 27}}\right)$

Solution :

$$= \frac{1}{\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}} = \frac{1}{3 \times 3} = \frac{1}{9}$$

Laws of exponent for rational exponents. If *x* and *y* are any rational numbers different from zero and *a, b* are any two integers then we have the following laws of rational exponents.

(i) $x^a \times x^b = x^{a+b}$ (ii) $\frac{x^a}{x^b} = x^{a-b}$ (iii) $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

Exercise 2.6

1. **Express each of the following in exponential form:**

(a) $\sqrt[5]{34}$ (b) $\sqrt[4]{27}$ (c) $\sqrt[11]{25}^{-2}$ (d) $\sqrt[7]{\frac{10}{7}}$

2. **Express each of the following as radicals:**

(a) $21^{1/8}$ (b) $27^{3/4}$ (c) $(335)^{7/5}$ (d) $\left(\frac{6}{19}\right)^{1/9}$

3. **Express each of the following with positive indices:**

(a) x^{-4} (b) $x^{-1/2}$ (c) $x^{-3/4}$ (d) $\frac{2}{5} x 1^{-7/8}$

4. **Simplify :**

(a) $x^{1/2} \times x^{3/2}$ (b) $\frac{x^{4/3}}{x^{1/3}}$ (c) $(x^{1/2})^4$ (d) $(x^5)^0$

5. **Evaluate the following :**

(a) $8^{2/3}$ (b) $27^{-2/3}$ (c) $\frac{1}{16^{-3/4}}$ (d) $\left(\frac{64}{729}\right)^{1/6}$

6. Determine x so that:- (a) $5^{1/3} \times 5^{1/6} = 5^{-x}$ (b) $800 = 8 \times 10^8 \times x^{-3/2}$.

7. By what number should we multiply $81^{3/16}$ so that the product becomes $3^{5/4}$?

8. If $4^x - 4^{x-1} = 24$ then find the value of $(2x)$.

9. Determine x and y so that $3^{x+y} = 81$ and $81^{x-4} = 3$.

10. **Evaluate the following**

(a) $(3^2+4^2)^{1/2}$ (b) $(5^2+12^2)^{1/2}$ (c) $\sqrt[3]{7} \times \sqrt[3]{49}$ (d) $(0.04)^{3/2}$

Radicals : If a is a rational number and n is a positive integer such that *the* n^{th} root of a i.e. $\sqrt{a}$ or $a^{1/n}$ is an irrational number then it is called $a^{1/n}$ radicals.

Example : $\sqrt{5}$ or $5^{1/2}$ since 5 is a rational number and 2 is a positive integer such that $5^{1/2}$ or $\sqrt{5}$ is an irrational number. So $\sqrt{5}$ is a radical of index 2.

Pure radical : A radical that contains no radical factor other than 1 is called a pure radical.

Example : $\sqrt{3}$ $\sqrt[5]{2}$ and $\sqrt[4]{3}$ are pure radicals.

Mixed Radicals : A radical which has a rational factor other than unity. The other factor being irrational is called a mixed radical.

Example : $5\sqrt[2]{3}$ $\sqrt[3]{12}$ and $2\sqrt[4]{5}$ are mixed radicals.

Simplest form of a square root radical : A square root radical is said to have in simplest form if –

(i) There is no fraction in the radical.

(ii) No perfect square is a factor of radical.

Example 6 : Express $\sqrt{\frac{125}{63}}$ in its simplest form:

Solution : we have $\sqrt{\frac{125}{63}} = \frac{\sqrt{125}}{\sqrt{63}}$

$$= \frac{\sqrt{5 \times 5 \times 5}}{\sqrt{3 \times 3 \times 7}} = \frac{\sqrt{5^2 \times 5}}{\sqrt{3^2 \times 7}}$$

$$= \frac{\sqrt{5^2}}{\sqrt{3^2}} \times \frac{\sqrt{5}}{\sqrt{7}} = \frac{5}{3} \times \frac{\sqrt{5}}{\sqrt{7}}$$

$$= \frac{5}{3} \times \sqrt{\frac{5 \times 7}{7 \times 7}} = \frac{5}{3} \times \frac{\sqrt{5 \times 7}}{\sqrt{7^2}}$$

$$= \frac{5}{3} \times \frac{\sqrt{35}}{7} = \frac{5}{21} \times \sqrt{35}$$

Example 7 : Simplify :

(i) $\sqrt{18}+\sqrt{50}-\sqrt{32}$ (ii) $\sqrt{84}\div\sqrt{7}$ (iii) $\dfrac{1}{6-\sqrt{3}}$

Solution : (i) $\sqrt{18}+\sqrt{50}-\sqrt{32} = \sqrt{9\times2} + \sqrt{25\times2} - \sqrt{16\times2}$

$= \sqrt{3^2\times2} + \sqrt{5^2\times2} - \sqrt{4^2\times2}$

$= 3\sqrt{2} + 5\sqrt{2} - 4\sqrt{2}$

$= (3+5-4)\sqrt{2} = 4\sqrt{2}$

(ii) $\sqrt{84}\div\sqrt{7} = \dfrac{\sqrt{84}}{\sqrt{7}} = \sqrt{\dfrac{84}{7}} = \sqrt{12} = \sqrt{2^2\times3} = 2\sqrt{3}$

(iii) $\dfrac{1}{6-\sqrt{3}} = \dfrac{1}{(6-\sqrt{3})}\times\dfrac{6+\sqrt{3}}{6+\sqrt{3}} = \dfrac{6+\sqrt{3}}{6^2-\left(\sqrt{3}\right)^2} = \dfrac{6+\sqrt{3}}{36-3} = \dfrac{6+\sqrt{3}}{33}$

Multiplying the numerator and denominator by $6+\sqrt{3}$

Exercise 2.7

1. Express the following radicals in exponential form:

(a) $\sqrt{5}$ (b) $\sqrt[3]{7}$ (c) $\sqrt[4]{\dfrac{3}{4}}$ (d) $\sqrt[8]{\dfrac{71}{2159}}$

2. Express the following as radicals in each case. Find the radical and the index:

(a) $16^{1/2}$ (b) $125^{1/3}$ (c) $\left(\dfrac{6}{17}\right)^{1/19}$

3. Express each of the following as mixed radicals :

(a) $\sqrt{18}$ (b) $\sqrt{405}$ (c) $\sqrt{108}$ (d) $\sqrt{300}$

4. Express each of the following as pure radicals :

(a) $2\sqrt{6}$ (b) $7\sqrt{5}$ (c) $4\sqrt{5}$ (d) $\dfrac{3}{2}\sqrt{\dfrac{3}{2}}$ (e) $10\sqrt{13}$

5. Express each of the following as a mixed radicals in the simplest form :

(a) $\sqrt{125}$ (b) $\sqrt{112}$ (c) $\sqrt{192}$ (d) $\sqrt{75}$

6. Simplify :

(a) $\sqrt{6}\times\sqrt{3}$ (b) $\sqrt{96}\div\sqrt{12}$ (c) $\sqrt{300}-\sqrt{48}+\sqrt{75}-\sqrt{27}$ (d) $(7\sqrt{2}+5)(7\sqrt{2}-5)$

7. Simplify :

(a) $\dfrac{\sqrt{126}\times\sqrt{63}\times\sqrt{45}}{\sqrt{147}\times\sqrt{243}}$ (b) $(\sqrt{5}+\sqrt{2})^2+(\sqrt{5}-\sqrt{2})^2$

Points to Remember :

- Exponents are powers to the numbers called bases.
- Very large and very small numbers are expressed in the form of exponents for convenience.
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$
- $\left(\frac{a}{b}\right)^{0} = 1$
- $\left(\frac{a}{b}\right)^{m} \times \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m+n}$
- $\left(\frac{a}{b}\right)^{m} \div \left(\frac{a}{b}\right)^{n} = \left(\frac{a}{b}\right)^{m-n}$ if $m>n$
- $\left(\frac{a}{b} \times \frac{c}{d}\right)^{n} = \left(\frac{a}{b}\right)^{n} \times \left(\frac{c}{d}\right)^{-n}$
- $\left(\frac{a}{b} \times \frac{c}{d}\right)^{-m} = \frac{1}{\left(\frac{a}{b} \times \frac{c}{d}\right)^{m}} = \left(\frac{b}{a} \times \frac{d}{c}\right)^{m}$
- $\left(\frac{a/b}{c/d}\right)^{n} = \frac{\left(\frac{a}{b}\right)^{n}}{\left(\frac{c}{d}\right)^{n}}$
- $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$
- A radical that contains no rational factors other than I is called a pure radical.
- A radical which has a rational factor other than unity. The other factor being irrational is called a mixed radical.

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs) :

Tick (✓) the correct options.

(a) Which of the following is correct for $(-1)^{20}$?

(i) -1 ☐ (ii) 0 ☐ (iii) 1 ☐ (iv) 20 ☐

(b) The reciprocal of $\frac{-2}{22}$ is equal to–

(i) $\frac{2}{22}$ ☐ (ii) $\frac{-2}{22}$ ☐ (iii) $\frac{21}{2}$ ☐ (iv) none of these ☐

(c) What is the value of 'm' for which $(-4)^{m+1} \times (-4)^{-2} = -64$?

(i) 4 ☐ (ii) -4 ☐ (iii) 3 ☐ (iv) 64 ☐

(d) What is the value of $11^{7/3} \div 11^{1/3}$?

(i) 111 ☐ (ii) 11 ☐ (iii) 144 ☐ (iv) 121 ☐

(e) $\frac{64}{343}$ Can be written as.

(i) $\frac{4^3}{7^2}$ ☐ (ii) $\left(\frac{4}{7}\right)^{-3}$ ☐ (iii) $\left(\frac{4}{7}\right)^{3}$ ☐ (iv) $\left(\frac{7}{3}\right)^{3}$ ☐

(f) The product of 4^5 and 4^3 is equal to –

(i) 16^5 ☐ (ii) 16^3 ☐ (iii) 4^2 ☐ (iv) 4^8 ☐

(g) What is value of 'x' in $3^{4x} = \frac{1}{81}$?

(i) 1 ☐ (ii) – 1 ☐ (iii) 0 ☐ (iv) – 2 ☐

(h) $2^9 \div 2^4$ is equal to –

(i) 2^5 ☐ (ii) 2^{13} ☐ (iii) 4^9 ☐ (iv) 4^4 ☐

2. Write the base and the exponent in each of the following :

(a) $\left(\frac{-1}{9}\right)^8$ (b) $(-18)^3$ (c) $(12)^{-15}$ (d) $\left(\frac{3}{19}\right)^{-3}$

(e) $(2^7)^{-6}$ (f) $\left\{\left(\frac{1}{3}\right)^{-2}\right\}^{-3}$ (g) $(2 \times 3)^2$ (h) $2^3 \div 2^2$

3. Simplify the following:

(a) $6^{15} \div 6^7$ (b) $\left(\frac{2}{5}\right)^3 \div \left(\frac{2}{5}\right)^5$ (c) $(ab)^5 \div (ab)$

(d) $\left(-\frac{3}{8}\right)^5 \div \left(-\frac{3}{8}\right)^7$ (e) $\left(\frac{1}{9}\right)^2 \times \left(\frac{1}{9}\right)^5$

4. Find the value of '*x*' in each of the following:

(a) $5^x = 125$ (b) $(2 \times 2)^x = 2^8$ (c) $\left(-\frac{2}{3}\right)^x = \frac{16}{81}$ (d) $(a^3 \times a^2) = a^x$

5. Simplify the following:

(a) $\frac{4^3 \times 5^a \times b^3}{4^2 \times 9^3 \times b^2}$ (b) $(6^\circ + 7^\circ)^2$ (c) $(2^\circ \times 3^\circ \times 4^\circ)^2$ (d) $\left(\frac{-1}{3}\right)^4 \times \left(\frac{-1}{3}\right)^5$

6. Find the value of *x* for each of the following:

(a) $x^5 \div x^3 = \frac{9}{16}$ (b) $\left(\frac{4}{15}\right)^3 \times \left(\frac{4}{15}\right)^{-6} = \left(\frac{4}{15}\right)^{2x+1}$

7. Find the value of p so that $\left(\frac{4}{5}\right)^3 \times \left(\frac{4}{5}\right)^{-3} = \left(\frac{4}{5}\right)^{3p}$

8. Simplify the following:

(a) $a^2 \times b^2$ (b) $\left(\frac{1}{3}\right)^3 \times \left(\frac{2}{3}\right)^3$ (c) $\left(\frac{1}{5}\right)^{-8} \times \left(\frac{5}{7}\right)^{-8}$ (d) $\left(\frac{1}{4}\right)^{-10} \times \left(\frac{2}{5}\right)^{-10}$

HOTS

1. The value of $(2^3)^2$ is equal to ....
2. The value of $\left(\frac{x}{y}\right)^0 \times \left(\frac{2}{3}\right)^3$ is ....

Lab Activity

Objective : To verify the law of exponents experimentally when the bases are different, i.e. $x^n \times y^n = (x \times y)^n$.

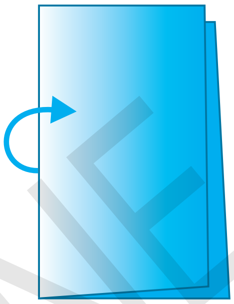
Materials Required : Glazed paper, white papers (to note the results), sketch pens, fevicol and chart paper.

Procedure :

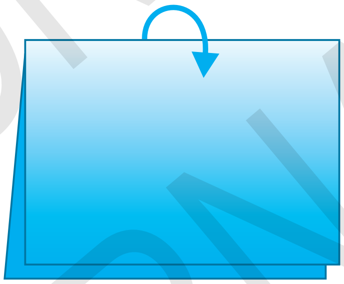
1. Take a glazed paper and fold it twice as shown.

Step 1:

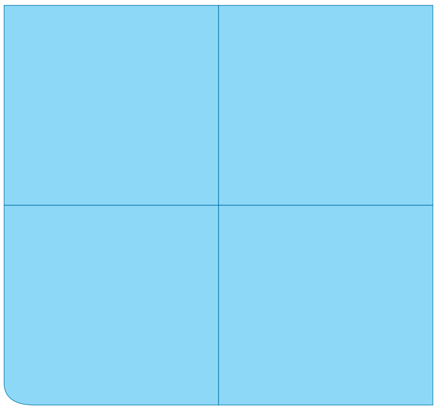
Step 2:



Step 3:



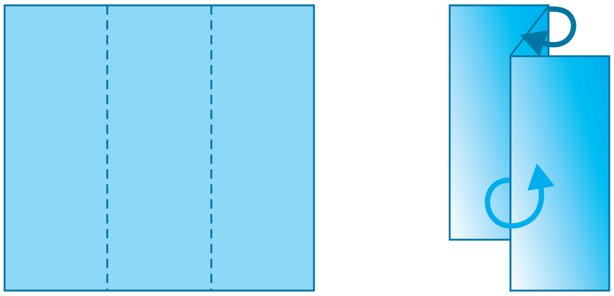
Step 4 : Unfold the paper



Now the creased paper represents $2^2 = 4$.

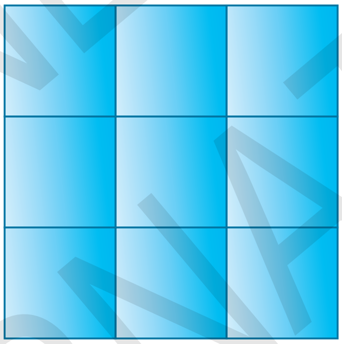
2. Take another paper and divide it into three parts, colour them and fold then as shown.

Step 1:



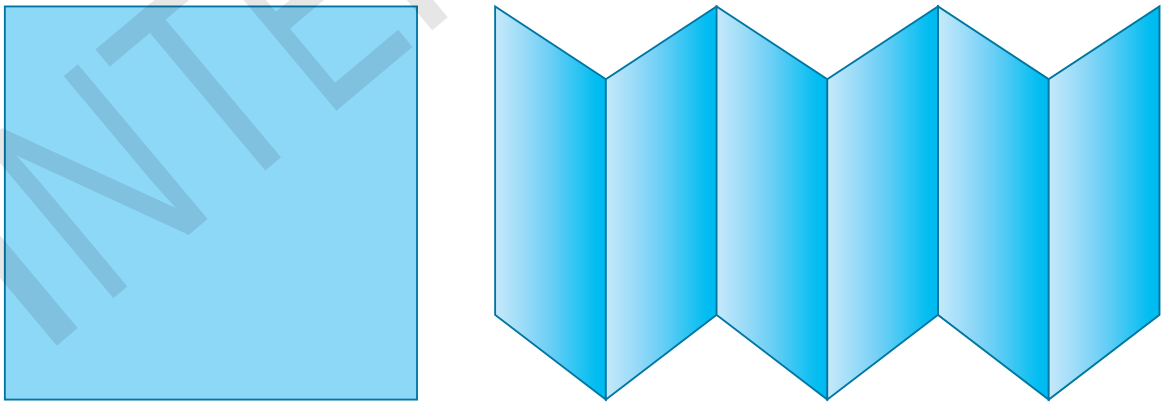
Step 2: Divide the folder paper again into three parts and fold again it as shown.

Step 3 : Unfold the paper.

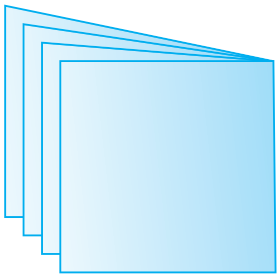


This creased paper represents $3^2 = 9$.

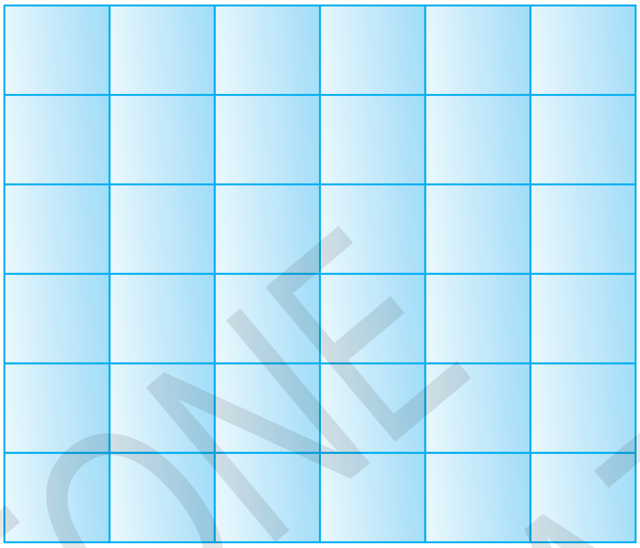
3. Take another paper and divide it into 6 parts.



Step 2 : Fold it as shown.



Step 3 : Unfold the paper



This creased paper represents $6^2 = 36$

So $2^2 \times 3^2 = 4 \times 9 = 36 = 6^2$

it is verified that $2^2 \times 3^2 = 36 = 6^2$

i.e. $x^n \times y^n = (x \times y)^n$