Contents

1 Knowing the Numbers

Recall what we have learnt in previous class about integers. We studied that integers comprise of whole numbers and negative numbers. We also learnt about the representation of integers on the number line, their comparative values, absolute value of an integer, addition and subtraction of integers. Let's briefly revise the main points again.

Integers

A combined set of negative numbers and whole numbers i.e, {........ –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ...}.
[In this set, all the natural numbers are positive integers and others are negative integers except '0' . 0 (zero) is neither positive nor negative.]

Absolute Value of an Integer

The absolute value of an integer is the magnitude of the numerical value of an integer regardless of its sign (direction). It is denoted by the symbol '| |'. The absolute value of an integer is always positive or 0 (zero).

Example : $|-5| = 5$

$|-6+4| = |-2| = 2$

$|7-3-4| = |0| = 0$

Also, the corresponding positive and negative integers have the same absolute value.

Example : $|25| = 25$ and $|-25| = 25$

Ordering of Integers

Integers and whole numbers obey the same rule in their ordering when represented on a number line, i.e. an integer represented to the right of any integer is greater than that integer and vice-versa.

Examples : $8 > 6, 5 > 3, 3 > 1, -1 > -3, -3 > -5, -7 > -8$

1. For every positive integer placed to the right of zero, there is a negative integer to the left of zero placed at the same distance from zero.
2. Zero is greater than every negative integer and less than every positive integer.
3. Every positive integer is greater than every negative integer.

Facts to Know

- The beginning of integers dates back to Babylonia around 4000 years back.
- The first evidence of the use of negative integers had been found in China around 300 BC.

Addition and Subtraction of Integers

1. When two integers with the same sign (either positive or negative) are added, their absolute values are added and the common sign is assigned to their sum.

$(+75)+(+25) = 100$

$(-35)+(-15) = -50$

2. When two integers with different signs are added, first the difference of their absolute values is found and it carries the sign of the integer having greater absolute value.

$(+30)+(-12) = 18$ $\quad$ $(-30)+(12) = -18$

$(+12)+(-30) = -18$ $\quad$ $(-12)+(+30) = 18$

3. When we have to subtract two integers, we change the integer to be subtracted into its corresponding opposite integer and then the two integer are simply added.

$(+18)-(+13)=(+18)+(-13)=5$ $\quad$ $(-18)-(-13)=(-18)+(+13)=-5$

$(+13)-(+18)=(+13)+(-18)=-5$ $\quad$ $(-13)-(-18)=(-13)+(+18)=5$

$(+18)-(-13)=(+18)+(+13)=31$ $\quad$ $(-18)-(+13)=(-18)+(-13)=-31$

$(+13)-(-18)=(+13)+(+18)=31$ $\quad$ $(-13)-(+18)=(-13)+(-18)=-31$

Example 1 : **Add the following :**

(a) (-30) and (-15) (b) (-6) and 18 (c) 4 and (-12) (d) (-52) and 25

Solution :

(a) (-30) and (-15) $= (-30)+(-15)$ $= -45$

(b) (-6) and 18 $= (-6)+18$ $= 12$

(c) 4 and (-12) $= 4+(-12)$ $= -8$

(d) (-52) and 25 $= (-52)+25$ $= (-27)$

Example 2 : **Subtract the following :**

(a) $(-8)-(-8)$ (b) $17-(-33)$ (c) $(-21)-(-8)$ (d) $20-(-14)$

Solution :

(a) $(-8)-(-8)$ $= -8+8=0$

(b) $17-(-33)$ $= 17+33=50$

(c) $(-21)-(-8)$ $= -21+8$ $= -13$

(d) $20-(-14)$ $= 20+14$ $= 34$

Exercise 1.1

1. **Arrange the following integers in ascending order :**
   (a) $-28, 7, 1, 3, -7, -12, -1$ (b) $11, -4, 8, 3, -5, 1, 6$
2. **Arrange the following integers in descending order :**
   (a) $-12, 15, 6, -14, -10, 3, -7, 9$ (b) $-16, 14, 2, 6, -8, -10, 0, 8, 13$
3. **Write the absolute value of the following :**
   (a) $|-27+68|$ (b) $|21-18|$ (c) $|-83+0|$ (d) $|72-99|$
4. **Add the following :**
   (a) $33+(-17)$ (b) $-180+212$ (c) $0+(-919)$ (d) $(-815)+(-913)$
5. **Subtract the following :**
   (a) $-32-(-48)$ (b) $(-45)-(+92)$ (c) $-8-(-15)$ (d) $165-(-319)$
6. **Which value is higher ?**
   (a) -4°C or 7°C (b) 0°F or -4°F (c) -17°C or -14°C (d) 44°F or 0°F
7. **Complete the following sequences :**
   (a) $-15, -12, -9, -6, -3,$ ____, ____, ____, ____ (b) 45, 51, 57, 63, ____, ____, ____, ____
8. **Write the opposites of the following integers :**
   (a) -15 $+17$ (b) -220 -110
   (c) -11 $+227$ (d) 0 $+1$
   (e) -1000 $+1$ (f) -99 -88

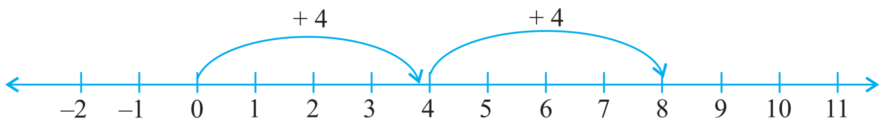
Multiplication of Integers

In previous classes, we have learnt that multiplication is nothing but repeated addition. Therefore, we can find the product of any two integers using repeated addition method.

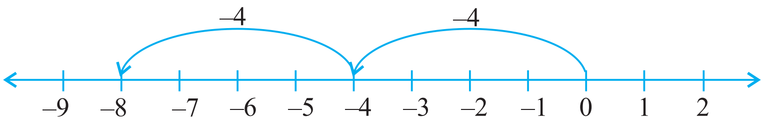
Example : $(+4) \times (+2) = (+4) \times 2 = (+4) + (+4) = 8$

$(-4) \times (+2) = (-4) \times 2 = (-4) + (-4) = -8$

On the number line, $(+4) \times (+2)$ means moving to the right of zero 2 times in steps of 4.



Similarly, $(-4) \times (+2)$ means moving to the left of zero 2 times in steps of 4.



Multiplication of Two Negative Numbers

Observe the following patterns :

$(-5) \times 4 = (-20)$
$(-5) \times 3 = (-15)$
$(-5) \times 2 = (-10)$
$(-5) \times 1 = (-5)$
$(-5) \times 0 = 0$

Here, we observe that when the multiplier is decreased by 1, the product increases by 5. Using this fact, we can proceed like this:

$(-5) \times (-1) = 5$
$(-5) \times (-2) = 10$
$(-5) \times (-3) = 15$
$(-5) \times (-4) = 20$

From the above given examples, we conclude that the product of two integers is the product of their absolute values and their signs are as follows :

(i) The product of two positive integers carries positive sign.
$(+9) \times (+8) = 72$
$(+12) \times (+7) = 84$

(ii) The product of two negative integers carries positive sign.
$(-15) \times (-6) = 90$
$(-9) \times (-8) = 72$

(iii) The product of two integers with opposite signs carries negative sign.
$(-17) \times (+4) = -68$
$(-7) \times (+13) = -91$

Facts to Know

- For multiplying integers we use the following rules :
  (i) Product of two positive integers is positive (i.e. $+ \times + = +$)
  (ii) Product of two integers with opposite sign is negative (i.e. $+ \times - = -$ or $- \times + = -$)
  (iii) Product of two negative integers is positive (i.e. $- \times - = +$)

Example 3 : **Find the product of the following :**

(a) $(-13)\times 9$ (b) $(-15)\times(-15)$ (c) $(+17)\times 5$ (d) $(+6)\times(-19)$

Solution :
(a) $(-13)\times 9 = -117$ (minus × plus = minus)
(b) $(-15)\times(-15) = 225$ (minus × minus = plus)
(c) $(+17)\times 5 = 85$ (plus × plus = plus)
(d) $(+6)\times(-19) = -114$ (plus × minus = minus)

Example 4 : **Find the value of the following :**

(a) $(-12+7)\times 5$ (b) $(-2)\times(-4)\times 4$ (c) $(-4-5)\times(-5)$ (d) $(-6+11)\times(-2+3)$

Solution :
(a) $(-12+7)\times 5$
$= (-5)\times 5$
$= -25$ [$-\times+=-$]

(b) $(-2)\times(-4)\times 4$
first we multiply -2 and -4
$(-2)\times(-4) = 8$ [$-\times-=+$]
Now, multiply
$8\times 4 = 32$

(c) $(-4-5)\times(-5)$
$= (-9)\times(-5)$
$= 45$ [$-\times-=+$]

(d) $(-6+11)\times(-2+3)$
$= 5\times 1$
$= 5$ [$+\times+=+$]

Division of Integers

It is well known that division is the inverse operation of multiplication. When we write $8 \times 4 = 32$, we can say $32 \div 4 = 8$ and $32 \div 8 = 4$. It means, for each multiplication statement there are two division statements.

So, when $(-7)\times 8 = -56$, then
$(-56)\div(-7) = 8$ and $(-56)\div 8 = -7$

Also ,when $(-4)\times(-7) = 28$, then
$28\div(-7) = -4$ and $28\div(-4) = -7$

Some more examples :

$(-5)\times 8 = (-40)$ So, $(-40)\div(-5) = 8$ and $(-40)\div 8 = (-5)$
$(-14)\times(-3) = 42$ So, $42 \div(-14) = -3$ and $42\div(-3) = (-14)$
$(-6)\times 8 = (-48)$ So, $(-48)\div(-6) = 8$ and $(-48)\div 8 = (-6)$
$4\times(-17) = (-68)$ So $(-68)\div 4 = -17$ and $(-68)\div(-17) = 4$

From the above examples, we can draw the following conclusions :

(i) When dividend and the divisor have the same sign (either both positive or both negative), the quotient carries a positive sign.

(ii) When dividend and divisor have different signs (one positive and other negative), the quotient carries a negative sign.

Facts to Know

- For dividing integers we use the following rules:
(i) $-\div-=+$ (ii) $+\div+=+$ (iii) $-\div+=-$ (iv) $+\div-=-$

Example 5 : **Determine the quotient for the following :**

(a) $(-76)\div(-19)$ (b) $75\div(-15)$ (c) $(-69)\div 23$ (d) $(-48)\div(-4)$

Solution : (a) $(-76) \div (-19)$

$= \frac{-76}{-19} = 4 \quad [\therefore - \div - = +]$

(b) $75 \div (-15)$

$= \frac{75}{-15} = (-5) \quad [\therefore + \div - = -]$

(c) $(-69) \div 23$

$= \frac{-69}{23} = (-3) \quad [\therefore - \div + = -]$

(d) $(-48) \div (-4)$

$= \frac{-48}{-4} = 12 \quad [\therefore - \div - = +]$

Example 6 : **Find the integer which gives (–54) when multiplied by (– 6).**

Solution : Let the unknown integer be x,

According to the question,

$x \times (-6) = (-54)$

$\Rightarrow x = \frac{-54}{-6} = 9 \quad [\ \ - \div - = +]$

Exercise 1.2

1. Multiply the following :

(a) 21 and 0 (b) (–12) and (–12) (c) (–192) and 0

(d) (–11) and (–12) (e) (–7) and 11 (f) $(-24) \times 5$

2. Find the product of each of the following :

(a) $16 \times (-5)$ (b) $(-6) \times (-3) \times (-3)$ (c) $5 \times 4 \times 3 \times 2 \times 0$

(d) $(-15) \times (-6)$ (e) $(-10) \times (-9) \times 5 \times 4$ (f) $(-10) \times (-10) \times 10$

(g) $(-2) \times (-5) \times (-6) \times 0$ (h) $2 \times 3 \times 4 \times (-5) \times (-5)$

3. Fill in the boxes for the following :

(a) $(-12) \times (-2) \times (-4) = \square$ (b) $(-2) \times (-2) \times 2 \times 2 = \square$ (c) $80 \div (-8) = \square$

(d) $\square \div (-25) = (-3)$ (e) $(-24) \div \square = (-6)$ (f) $123 \div \square = (-41)$

(g) $66 \div \square = (-11)$ (h) $84 \div (-12) = \square$ (i) $[(-2) \times \square] \div 6 = -1$

(j) $(-5) \times (-2) \times 10 = \square$

4. Determine the quotient for each of the following :

(a) $(-20) \div (-1)$ (b) $(-72) \div 6$ (c) $(-26) \div 13$

(d) $36 \div (-3)$ (e) $(-126) \div 6$ (f) $(-111) \div (-3)$

(g) $(-117) \div (-13)$ (h) $(-14) \div (-14)$ (i) $(-81) \div 9$

(j) $(-100) \div (-10)$

5. Simplify and find the absolute value of the following :

(a) $|(-5 + 6 - 5) \times (-3)|$ (b) $|(-18) \div 3|$ (c) $|\{(-8) - (-4)\} \div (-2)|$ (d) $|(-5) \times (-5) \times (-2) \times 2|$

6. Find the integer which gives (–51) when it is multiplied by (–17).

7. Find the quotient when divisor and dividend are (–4) and (–48) respectively.

8. An integer when divided by –7 gives 12. Find the integer.

Properties of Integers

Properties of Addition

Closure Property **:** If a and b are two integers, then a + b will always be an integer.

Examples : (–7) + (–8) = – 15

(–2) + 4 = 2

18 + (–12) = 6

(–16) + (–12) = –28

Commutative Property **:** If a and b are two integers, then their sum remains the same, irrespective of the order, i.e. a + b = b + a

Examples : (–12) + (–16) = (–16) + (–12) = (–28)

(–5) + 47 = 47 + (–5) = 42

10 + (–65) = (–65) + 10 = (–55)

(–17) + (–18) = (–18) +(–17) = –35

Associative Property **:** If a, b and c are three integers, then a + (b + c) = (a + b) + c. While adding these integers it is not necessary to add it in a particular order of their occurence. We can do it by grouping them as per our convenience.

Examples : [(–8) + 7] + (–15) = (–8) + [7 + (–15)]

⇒ (–1) + (–15) = (–8) + (–8)

⇒ (–16) = (–16)

Let's see another example :

[(–7) + (–8)] + (–11) = (–7) + [(–8) + (–11)]

⇒ (–15) + (–11) = (–7) + (–19)

⇒ (–26) = (–26)

Additive Identity **:** If 0 (zero) is added to any integer, its value remains same. Thus, for an integer 'a', a + 0 = 0 + a = a

Examples : (–29) + 0 = (–29)

0 + (–35) = (–35)

Additive Inverse **:** The sum of an integer and its opposite is always zero. If 'a' is an integer, then (–a) is its opposite and vice versa such that a + (–a) = 0 = (–a) + a

Examples : 29 + (–29) = 0 = (–29) + 29

(–18) + 18 = 0 = 18 + (–18)

(–95) + 95 = 0 = 95 + (–95)

In the above given examples, the integers of each pair, i.e. (29, –29), (18, –18) and (95, –95) are additive inverse of each other.

Property of 1 **:** If 1 is added to any integers, it gives its successor.

Examples : 11 + 1 = 12

So, 12 is the successor of 11.

Also ,

(–7) + 1 = (–6) So, – 6 is the successor of (–7).

Properties of Subtraction

Closure Property **:** If a and b are two integers, then a – b will always be an integer.

Examples : (–4) – (–8) = 4

(–8) – 12 = (–20)

4 – 11 = (–7)

(–17) – (–14) = (–3)

Commutative Property **:** If a and b are two integers, then, a – b ≠ b – a. It means commutative property is not applicable for the subtraction of integers.

Examples : (–7) – (–4) = (–3) but (–4) – (–7) = 3

11 – (–17) = 28 but (–17) – 11 = (–28)

(– 4) – 18 = (–22) but 18 – (–4) = 22

Associative Property **:** If a, b and c are three integers, then (a – b) – c ≠ a – (b – c). It means associative property is not applicable for the subtraction of integers.

Examples : [(–2) –4] – (–7) ≠ (–2) – [4 – (–7)]

⇒ (–6) – (–7) ≠ (–2) – 11

⇒ 1 ≠ –13

Property of Zero **:** If 0 (zero) is subtracted from any integer, its value remains same. Thus, for an integer 'a', a – 0 = a

Examples : (–32) – 0 = (–32)

(–75) – 0 = (–75)

99 – 0 = 99

Property of 1 **:** If 1 is subtracted from any integer, it gives its predecessor.

Examples : (–23) –1 = (–24)

83 – 1 = 82

(–87) – 1 = (–88)

In the above given examples, the integers (– 24), 82 and (–88) are predecessors of (–23), 83 and (–87) respectively.

Properties of Multiplication

Closure Property **:** If a and b are two integers, then a × b will always be an integer.

Examples : (–7) × (–8) = 56

(–8) × 4 = (–32)

7 × (–3) = (–21)

11×12 = 132

Commutative Property **:** If a and b are two integers, then their multiples remains the same, irrespective of the order, i.e. a × b = b × a

Examples : (–7) × (–12) = 84 = (–12) × (–7)

4 × (–9) = (–36) = (–9) × 4

(–13) × 6 = (–78) = 6 × (–13)

17 × 5 = 85 = 5 × 17

Associative Property **:** If a, b and c are three integers, then a × (b × c) = (a × b) × c. While multiplying these integers it is not necessary to multiply it as a particular order of their occurrence. We can do it by grouping them as per our convenience.

Examples : $[(-5)\times 6]\times(-7) = (-5)\times[6\times(-7)]$

$(-30)\times(-7) = (-5)\times(-42)$

$\Rightarrow\ 210 = 210$

Let's see another example :

$[(-12)\times(-5)]\times(-2) = (-12)\times[(-5)\times(-2)]$

$\Rightarrow\ 60\times(-2) = (-12)\times 10$

$\Rightarrow\ -120 = -120$

Multiplicative Identity : If 1 is multiplied to any integer, its value remains same. Thus, for an integer 'a'

$$a\times 1 = 1\times a = a$$

Examples : $(-87)\times 1 = 1\times(-87) = (-87)$

$43\times 1 = 1\times 43 = 43$

$(-27)\times 1 = 1\times(-27) = (-27)$

Multiplicative Inverse : The multiple of an integer and its multiplicative inverse is always 1. If 'a' is an integer, then $\left(\frac{1}{a}\right)$ is its multiplicative inverse and vice-versa such that

$$a\times\frac{1}{a} = 1 = \frac{1}{a}\times a$$

Examples : $(-15)\times\frac{1}{(-15)} = 1 = \frac{1}{(-15)}\times(-15)$

$(-43)\times\frac{1}{(-43)} = 1 = \frac{1}{(-43)}\times(-43)$

Distributive Property : If a, b and c are three integers, then

$$a\times(b+c) = (a\times b)+(a\times c)$$

It means multiplication distributes over addition.

Examples : $(-7)\times[4+(-8)] = [(-7\times 4)]+[(-7)\times(-8)]$

$\Rightarrow\ (-7)\times(-4) = -28+56$

$\Rightarrow\ 28 = 28$

Facts to Know

- 1 and –1 are the only two integers whose respective multiplicative inverse remains same.

Properties of Division

Closure Property : If a and b are two integers, thus $a\div b$ will not always be an integer.

Examples : $(-2)\div 5 = -0.4$

$2\div 3 = 0.\overline{6}$

$(-4)\div 0 =$ not defined

$(-7)\div(-28) = 0.25$

Commutative Property : If a and b are two integers, then $a\div b \neq b\div a$. It means commutative property is not applicable for the division of integer.

Examples : $(-4)\div(8) \neq 8\div(-4)$

$\Rightarrow\ -0.5 \neq -2$

Let's see another example:

$(-12) \div (-4) \neq (-4) \div (-12)$

$\Rightarrow 3 \neq 0.\overline{3}$

Associative Property : If a, b and c are three integers, then $(a \div b) \div c \neq a \div (b \div c)$. It means associative property is not applicable for the division of integers.

Examples : $[(-16) \div 4] \div (-2) \neq (-16) \div [4 \div (-2)]$

$\Rightarrow (-4) \div (-2) \neq (-16) \div (-2)$

$\Rightarrow 2 \neq 8$

Let's see another example:

$[(-24) \div (-4)] \div 6 \neq (-24) \div [(-4) \div 6]$

$\Rightarrow 6 \div 6 \neq (-24) \div \left(\frac{-4}{6}\right)$

$\Rightarrow 1 \neq 36$

Property of 1 : If we divide any integer by 1, it gives the same integer as quotient. For any integer 'a', $a \div 1 = a$

Examples : $(-29) \div 1 = (-29)$

$(-75) \div 1 = (-75)$

Property of 0 (zero) : If we divide zero by any integer, the result is always zero. For any integer 'a',

$0 \div a = 0$

Example : $0 \div -92 = 0$

$0 \div 345 = 0$

$0 \div 645 = 0$

Facts to Know

- Zero divided by an integer is always zero.
- Any integer divided by zero is not defined.

Example 7 : Sum of two integers is (–73). If one of the integers is 46, find the other integer. Also verify the answer.

Solution : Let the required integer be 'x'

According to the question,

$x + 46 = (-73)$

$\Rightarrow x = (-73) - 46$

$\Rightarrow x = -119$

Hence, the required integer is (–119)

Verification :

$-119 + 46 = -73$

$\Rightarrow -73 = -73$

Example 8 : Subtract the sum of (–75) and (– 47) from the multiple of (–8) and (–12).

Solution : Sum of (–75) and (– 47)

$= (-75) + (-47)$

$= -122$

Multiple of (–8) and (–12)

$= (-8) \times (-12) = 96$

$\therefore$ The required difference $= 96 - (-122) = 218$

Example 9 : **Subtract the sum of 1 6, 14 and (–7) from the sum of 7, 19 and the additive inverse of 16.**

Solution : Sum of 16, 14 and (–7)

$= 16 + 14 + (-7) = 23$

Additive inverse of 16 is (–16),

Sum of 7, 19 and (–16)

$= 7 + 19 + (-16) = 10$

$\therefore$ The required difference $= 10 - 23 = (-13)$

Example 10 : **In a quiz competition, Rahul scored 70, (–20) and 40, whereas Saurav scored 50, 0 and 20 in three successive rounds. Who won the quiz competition and by what margin ?**

Solution : Rahul's total score $= 70 + (-20) + 40 = 90$

Saurav's total score $= 50 + 0 + 20 = 70$

Rahul's score is 90 and Saurav's score is 70.

So, Rahul won by the margin of $90 - 70 = 20$.

Example 11 : **Evaluate the following using distributive property :**

(a) $(-6) \times 97$ (b) $(-7) \times 1002$

Solution : (a) $(-6) \times 97 = (-6) \times (100 - 3)$

$= [(-6) \times 100] - [(-6) \times 3]$ (by distributive property)

$= (-600) - (-18)$

$= (-600) + 18 = (-582)$

(b) $(-7) \times 1002 = (-7) \times (1000 + 2)$

$= [(-7) \times 1000] + [(-7) \times 2]$

$= (-7000) + (-14) = (-7014)$

Example 12 : **A submarine descends into sea water at the rate of 12 km per minute. What will be its position after 15 minutes ?**

Solution : Since the submarine is going down, so the distance covered by it will be represented by negative integer.

Change in position of the submarine in one minutes $= (-12)$ km

Position of the submarine after 15 minutes $= (-12) \times 15 = (-180)$ km

It means, after 15 minutes the submarine will be 180 km below the surface of sea water.

Example 13 : **A room heater starts raising the temperature at the rate of 2°C per minute. Ankit feels comfortable at 34°C. In how much time will Ankit be happy ? [If current temperature is 0°C]**

Solution : Times taken by room heater to reach at 34°C $= \frac{34}{2} = 17$ min

Hence, after 17 minutes the temperature of the room will reach at 34°C that will make Ankit happy.

Exercise 1.3

1. **Fill in the blanks :**

(a) $20 \div \square = -1$ (b) $\square \div 25 = -1$ (c) $\square \div 6 = 0$

(d) $13 - (-13) = \square$ (e) $(-17) \times \square = 16 \times (-17)$ (f) $(-243) \times 0 =$

(g) $(-5) \times \frac{1}{\square} = 1$ (h) $(-10) \times [(-5) + 7] = [(-10) \times \square] + [(\square) \times 7]$

(i) $\square \times [3 + (-4)] = [(-8) \times 3] + [(-8) \times \square]$

(j) $[10 \times \square] \times (-6) = 10 \times [(-5) \times (-6)]$

2. **Write T for True and F for False for the following statements :**

(a) $(-5) \times 7 = -35$ $\square$ (b) $(-5) \times (-4) \times 2 = 5 \times 2 \times (-4)$ $\square$ (c) $(-4) - (-8) = -12$ $\square$

(d) $(-18) \times 0 = -18$ $\square$ (e) $(-8) \times (-14) \times 0 = 0$ $\square$ (f) $(-12) \div 4 = 4 \div (-12)$ $\square$

3. **Calculate the following using suitable arrangements:**

(a) $(127 \times 8) + (127 \times 12)$ (b) 105×7 (c) $(-125) + (-75) + 188 + (-38) + 25$

4. **Prove the validity of the following using property of multiplication:**

(a) $[57 \times (-127)] + [(-23) \times 57]$ (b) $(-8) \times [(-12) + (-16)]$

(c) $[(-21) \times [(-5) + (-7)]$ (d) $[(-105) \times 91] + [5 \times 91]$

5. **Evaluate the following using distributive property:**

(a) 18×102 (b) $(-5) \times 1003$ (c) 89×14 (d) $(-37) \times 97$

6. An integer divided by (-13) gives 9. Find the integer.

7. Find an integer which when multiplied with (-8) gives 72.

8. Tanya throws a stone 52 m vertically up in the air which later settled on the bottom of a lake 60 m deep. Find the total distance covered by stone.

Points to Remember

- Integers are the combined sets of negative numbers and whole numbers.
- The absolute value of an integer is the right magnitude of the value of an integer regardless of its sign.
- On a number line, every integer to the right of zero is greater than the integers to its left and vice versa.
- 0 is neither negative nor positive.
- Addition of integers follow closure, commutative and associative property i.e.,
  (a) The value of sum of two integers a and b is integer. (b) $a + b = b + a$
  (c) $(a + b) + c = a + (b + c)$
- Subtraction of integers follow only closure property. The value of subtraction of two integers a and b is integer.
- Multiplication of integers follow closure, communicative and associative property i.e.,
  (a) The product of two integers a and b is integer. (b) $a \times b = b \times a$ (c) $(a \times b) \times c = a \times (b \times c)$
- Integers follow distributive property, i.e. for any three integers a, b and c,
  $a \times (b + c) = (a \times b) + (a \times c)$
- For division of integers
  (a) The quotient carries a positive sign when dividend and divisor have same sign (either positive or negative).
  (b) The quotient carries a negative signs when dividend and divisor have different signs (one positive and other negative).
- For any integer 'a',
  (a) $a \div 0$ is not defined (b) $0 \div a = 0$ and (c) $a \div 1 = a$

EXERCISE

1. MULTIPLE CHOICE QUESTIONS (MCQs):

Tick (✓) the correct options :

(a) Which of the following is correct ?

(i) $a \div 0 = 0$ (ii) $0 \div a =$ not defined

(iii) $0 \div a = 0$ (iv) $1 \div a = a$

(b) The smallest positive integer is :

(i) 0 (ii) 1 (iii) 99 (iv) 1000

(c) The additive inverse of (–7) is :

(i) $\frac{1}{7}$ (ii) $-\frac{1}{7}$ (iii) 7 (iv) None of these

(d) The multiplicative inverse of 7x is :

(i) $\frac{1}{7x}$ (ii) $\frac{7}{x}$ (iii) $\frac{7x}{1}$ (iv) (–7x)

(e) Which of the following relation is correct ?

(i) $(-24) \div (-4) = (-6)$ (ii) $(-72) \div 8 = (-9)$

(iii) $20 \div (-10) = 2$ (iv) $(-30) \div (-3) = -27$

(f) What is the value of expression:

$\{(-15) + 4 + (-7)\} \div \{(-3) - 9 + 18\}$

(i) 2 (ii) –3 (iii) 4 (iv) 6

(g) The expression $103 \times (-35)$ can be re-written as which of the following expression ?

(i) $10300 - 105$ (ii) $-10300 + 105$

(iii) $3500 + 105$ (iv) $-3500 - 105$

2. Find the product of the following :

(a) $(-13) \times (-11)$ (b) $(-4) \times 16$ (c) $(-6) \times (-15)$ (d) $(-125) \times 8$

3. Find the quotient :

(a) $(-64) \div (-16)$ (b) $(-84) \div 4$ (c) $0 \div (-7)$

(d) $168 \div (-8)$ (e) $(-216) \div (-6)$ (f) $120 \div (-15)$

4. Simplify :

(a) $\{(-80) \div 16\} \times (-2) + 6$ (b) $[\{(-3) - (-4)\} \times \{(-6) + (-8)\}]$

(c) $\{(-7) \times (-5) \times (-6)\} \div (-15)$ (d) $[24 \div \{(-6 + 18) - 6\}]$

5. Find the product using suitable properties as:

(a) $(-17) \times 29$ (b) $(-51) \times 13$

(c) $\{(-47) \times 21\} + (-47)$ (d) $[(-24) \times 37] + [(-24) + 13]$

(e) $(99 \times 17) + (17 \times 101)$ (f) $(-12) \times 25 \times 6 \times (-4)$

6. A monkey is trying to reach at the top of vertical slipperly pole 25 m high. In one jump he goes up by 3 m but slips by 1 m. In how many moves he will reach at the top?

7. Rinku gets ₹ 1500 as pocket money per month. He spends ₹ 750 eating outside and ₹ 300 as donation to a charity organisation. Find out Rinku's saving at the end of the year.

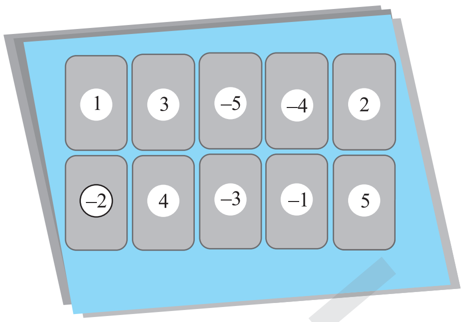
HOTS

Every floor of a 20 storey building is 5 m high. If a lift moves 2 metres every second, how long will it take to move from 3rd floor to 15th floor.

Lab Activity

Objective : To comprehend the operation of multiplication through a game.

Materials Required : 10 square-shaped one coloured cards with integers from –5 to 5 mentioned on it, square paper sketch pens and different coloured counters.



Procedure : The game is to be played ideally among 2 to 4 players.

Step 1 : Take a white coloured chart paper. Cut it in squared shape.
Take a chart paper and draw a 9 × 9 grid on it using sketch pen.

32	33	34	35	36	37	38	39	40
31	30	29	28	27	26	25	24	23
14	15	16	17	18	19	20	21	22
13	12	11	10	9	8	7	6	5
–4	–3	–2	–1	0	1	2	3	4
–5	–6	–7	–8	–9	–10	–11	–12	–13
–22	–21	–20	–19	–18	–17	–16	–15	–14
–23	–24	–25	–26	–27	–28	–29	–30	–31
–40	–39	–38	–37	–36	–35	–34	–33	–32

Step 2 : Write the integers starting from – 40 to 40 as shown.

Step 3 : Keep the counters at 0. [Each player will play with only counter].

Step 4 : Rules to be followed:
Each player will choose two cards from the pack of 10 cards randomly. (Integers hidden on other side)
The player has to multiply the integers shown on the card. For example, the appearing numbers are (–4) and 5. So, (–4)×5=(–20). The player will put his counter at (–20) on the board.

Step 5 : Now its turn of the next player sitting next to the first player.

Step 6 : If the product is positive, the counter will move towards 40. If the product is negative it will move towards (–40).

Step 7 : Each player gets another chance to choose two cards after the completion of the first round.

Step 8 : The player whose counter crosses 40 first is the winner.

2 Rational Numbers

When one starts learning Mathematics, he first starts with counting numbers, i.e., 1, 2, 3, 4, 5, 6 ..... (Natural Numbers). Then he learns about 0 and numbers beyond 1 (Whole Numbers). Knowing about the negatives of natural numbers he learns about integers. Suppose one has to find out a part of a whole or a part of a set of objects, he may or may not get the desired result. Therefore, we need to extend the set of whole numbers so as to make the division of whole always possible. This new set of numbers is termed as the set of **rational numbers.** Here, rational number is any number that can be written as a **ratio of two integers.** It means a number is rational, if it can be written as a fraction where both numerator and denominator are integers.

The word 'rational' originates from the word 'ratio' because rational numbers are the ones that can be written in ratio form, $\frac{p}{q}$ where p and q are integers and $q \neq 0$.

Before knowing more about rational numbers, let's refresh the basics of number system.

Natural numbers : The counting numbers are called natural numbers. Thus, 1, 2, 3, 4, 5 ..... are natural numbers.

Whole Numbers : All natural numbers including 0 (zero) are called whole numbers. Thus, 0, 1, 2, 3...... are whole numbers.

We also learnt these properties related with whole numbers:

(a) The sum of two whole numbers is always a whole number.

Example : $0 + 8 = 8$

$17 + 25 = 42$

$29 + 97 = 126$

(b) The product of two whole numbers is always a whole number.

Example : $0 \times 17 = 0$

$8 \times 15 = 120$

$7 \times 19 = 133$

Division of integers by zero is not defined.
i.e 7 ÷0 = 0
(–6) ÷ 0 = 0

(c) The difference of two whole numbers is not always a whole number.

Example : $57 - 30 = 27$

$0 - 68 = (-68)$ (Not a whole number)

$75 - 87 = (-12)$ (Not a whole number)

(d) The division of two whole numbers does not always give a whole number as quotient.

Example: $12 \div 3 = 4$

$14 \div 6 = 2.\overline{3}$ (Not a whole number)

$49 \div 7 = 7$

Integers : The whole numbers with the negatives of natural numbers are called Integer.

Thus, ... –4, –3, –2, –1, 0, 1, 2, 3, 4, ... are all integers.

We learnt from above that subtraction of whole numbers may give us integers.

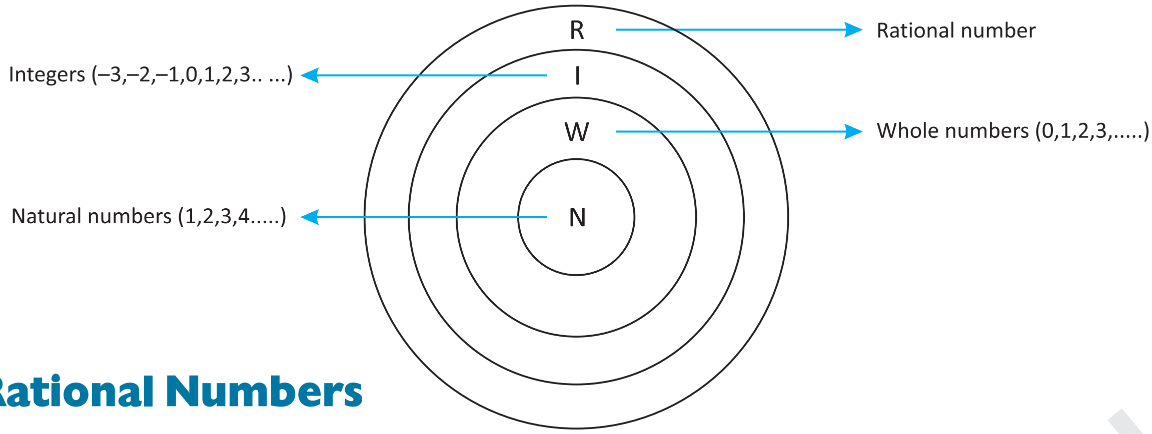
Also, if you recall the previous chapter, while dealing with properties of integers, we saw that the division of one integer by another integer may or may not be an integer.

Example : $24 \div 8 = 3$

$(-72) \div 18 = (-4)$

$(-5) \div 15 = \frac{-1}{3}$ [Not an integer]

$(-3) \div (-18) = \frac{1}{6}$ [Not an integer]



Rational Numbers

Therefore, we need to extend our number system in which negative and positive fractions must be included. This new system of numbers is known as rational number system.

A number that can be represented in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is called a rational number.

The denominator of a fraction cannot be 0 (zero) because division of any number by zero is not defined.

Examples : (a) $\frac{-6}{11}$ is a rational number as both (–6) and 11 are integers and denominator (11) is not equal to 0.

(b) $\frac{-3}{8}$ is a rational number as both (–3) and 8 are integers and denominator (8) is not equal to zero.

(c) 15 is a rational number as $15 = \frac{15}{1}$.

Both 15 and 1 are integers and the denominator (1) is not equal to 0 (zero).

From the above, we can conclude that rational numbers include both integers and fractions.

Facts to Know

- If both the numerator and denominator of a rational number are either both positive or both negative, it is a positive rational number.

Types of Rational Numbers

There are two types of rational numbers:

(a) **Positive Rational Numbers** (b) **Negative Rational Numbers**

(a) **Positive Rational Numbers :** A rational number is positive, if its numerator and denominator are either both positive or both negative.

Example : $\frac{2}{7}, \frac{8}{19}, \frac{23}{5}, \frac{0}{4}, \frac{7}{9}$ are positive rational integers

Also, $\frac{-7}{-18}, \frac{-2}{-9}, \frac{-13}{-77}, \frac{-1}{-6}$ are positive rational numbers, because $\frac{-7}{-18} = \frac{7}{18}, \frac{-2}{-9} = \frac{2}{9}, \frac{-13}{-77} = \frac{13}{77}$ *and* $\frac{-1}{-6} = \frac{1}{6}$

(b) Negative Rational Numbers : A rational number is negative, if either its numerator or denominator is negative.

Example : $\frac{-3}{7}, \frac{3}{-17}, \frac{18}{-29}, \frac{-37}{39}$ are negative rational numbers because

$$\frac{-3}{7} = \left(-\frac{3}{7}\right), \frac{3}{-17} = \left(-\frac{3}{17}\right), \frac{18}{-29} = \left(-\frac{18}{29}\right), \frac{-37}{39} = \left(-\frac{37}{39}\right)$$

Facts to Know

- A rational number $\frac{p}{q}$ is a fraction only when p and q are whole numbers and $q \neq 0$.
- All fractions are rational numbers but a rational number is not always a fraction.
- Negative fractions are also called negative rational numbers.
- All counting numbers (Natural numbers), whole numbers and integers are rational numbers.

Rational Numbers on a Number Line

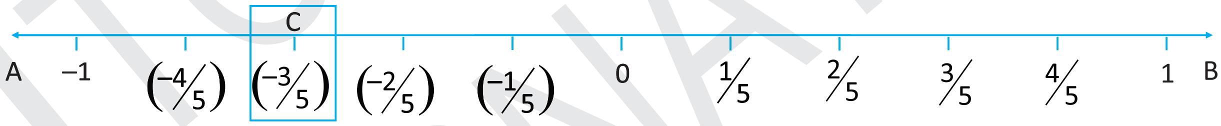
We already know how to represent fractions, integers, whole numbers and natural numbers on number line. Let's learn to denote rational numbers on a number line.

When we move from left to right on the number line, the number increases, whereas, if we move from right to left, the number decreases as shown in the given figure.

To represent a rational number on a number line, each unit length is divided into equal parts of the denominator of a rational number. Then one can easily mark the required rational number on line.

Example 1 : Represent $\frac{-3}{5}$ on a number line.

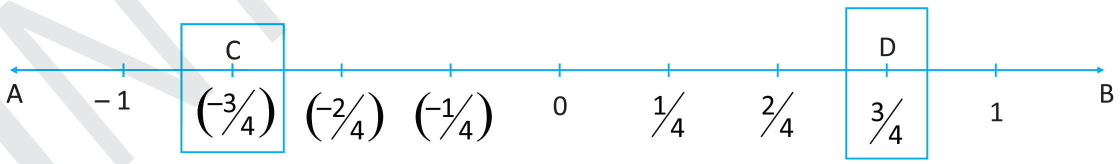
Solution : Here, the denominator of the rational number is 5. So we divide each unit length on the number line AB into 5 equal parts as shown is the figure.



The numerator to be denoted, is (–3), so counting 3 parts to the left of zero on the number line, mark it as point C. Point 'C' represents $\left(\frac{-3}{5}\right)$.

Example 2 : Represent $\left(\frac{-3}{4}\right)$ and $\frac{3}{4}$ on a number line.

Solution : Here, the denominator of the rational number is 4. So we divide each unit length on the number line AB into 4 equal parts as shown in the figure.

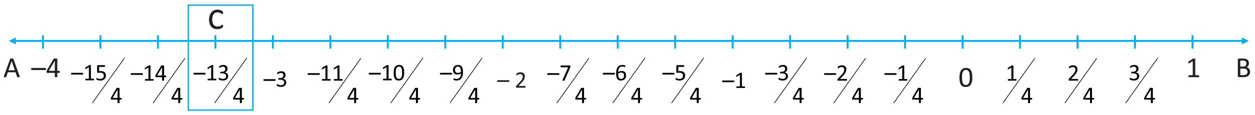


The numerators to be denoted are (–3) and 3. So counting 3 parts to the left of zero on the number line, mark it as point 'C' and counting 3 parts to the right of zero on the number line, mark it as point D. Point C and D represents $\left(\frac{-3}{4}\right)$ and $\frac{3}{4}$ respectively.

Example 3 : **Represent $\frac{-13}{4}$ on a number line.**

Solution : $\frac{-13}{4} = -3\frac{1}{4} = -3 + \left(\frac{-1}{4}\right)$

Here, the denominator of the rational number is 4. So we divide each unit length on the number line AB into 4 equal parts as shown in the figure.



Since the numerator to be denoted is –13, so count 13 parts to the left of zero on the number line and mark it as point 'C'. Point 'C' represents $\left(\frac{-13}{4}\right)$.

Standard Form of a Rational Number

A rational numbers is said to be in the standard form if its denominator is a positive integer and the numerator and the denominator are co-primes i.e., have no common factor other than 1.

Example : $-\frac{2}{7}, \frac{17}{47}, \frac{8}{19}$ and $\frac{-17}{6}$ are the rational numbers in standard form.

Example 4 : **Reduce the following in the standard form :**

(a) $\frac{-4}{26}$ (b) $\frac{24}{-16}$

Solution : (a) The denominator of $\left(\frac{-4}{26}\right)$ is positive. To express it in standard form, first find the HCF of 4 and 26, which is 2. Now denominator is positive.

now $\frac{-4}{26} = \frac{(-4) \div 2}{26 \div 2}$

$= \frac{-2}{13}$

So, the standard form of $\left(\frac{-4}{26}\right)$ is $\left(\frac{-2}{13}\right)$.

(b) The denominator of $\left(\frac{24}{-16}\right)$ is negative. To make it positive, multiply both numerator and denominator by (–1), we get

$\frac{(24) \times (-1)}{(-16) \times (-1)} = \frac{-24}{16}$

Now find the HCF of 24 and 16, which is 8. Divide both numerator and denominator by 8, we get

$\frac{(-24) \div 8}{16 \div 8} = \frac{-3}{2}$

So, the standard form of $\left(\frac{24}{-16}\right)$ is $\left(\frac{-3}{2}\right)$.

Absolute Value of a Rational Number

The absolute value of a rational number is its positive numerical value, irrespective of the sign of numerator and denominator, i.e., $\left|\frac{p}{q}\right| = \frac{p}{q}$, $\left|\frac{-p}{q}\right| = \frac{p}{q}$, $\left|\frac{p}{-q}\right| = \frac{p}{q}$ and $\left|\frac{-p}{-q}\right| = \frac{p}{q}$

Example : (a) $\left|\frac{-7}{-6}\right| = \frac{7}{6}$ (b) $\left|\frac{-17}{18}\right| = \frac{17}{18}$

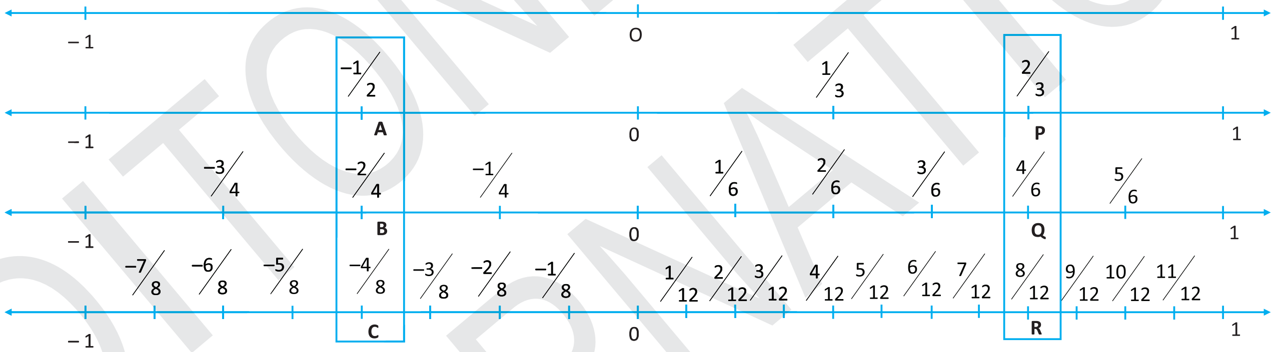
(c) $\left|\frac{27}{8}\right| = \frac{27}{8}$ (d) $\left|\frac{6}{-13}\right| = \frac{6}{13}$

Facts to Know

- The absolute value of rational number, when represented on a number line, is taken as its distance from zero (irrespective of the direction) and it is represented as $\frac{p}{q}$.
- A positive rational number is always greater than a negative rational number.

Equivalent Rational Numbers

Let's understand the concept of equivalent rational numbers through number lines. Draw the number lines as shown in the figure.



One can observe that points P, Q and R representing $\frac{2}{3}, \frac{4}{6}$ and $\frac{8}{12}$ respectively are equidistant from point O that represents (0) zero on the number line. In other words, the same point corresponds to these rational numbers. We can say rational numbers $\frac{2}{3}, \frac{4}{6}$ and $\frac{8}{12}$ are equivalent. Similarly points A, B and C, representing $\left(\frac{-1}{2}\right), \left(\frac{-2}{4}\right), \left(\frac{-4}{8}\right)$ and respectively are equidistant from point O that represents 0 on the number line. Hence, these are equivalent,

i.e., $\frac{-1}{2} = \frac{-2}{4} = \frac{-4}{8}$

Thus, rational numbers which can be represented by the same point on a number line are called equivalent rational numbers.

Let $\frac{a}{b}$ and $\frac{c}{d}$ be two rational numbers:

(a) If $\frac{a}{b} = \frac{c}{d}$ or ad = bc, they are equivalent rational numbers.

(b) If $\frac{a}{b} > \frac{c}{d}$, then ad > cd (c) if $\frac{a}{b} < \frac{c}{d}$, then ad < bc

Equivalent rational numbers can be obtained by multiplying or dividing both the numerator and the denominator of the given rational number by the same non-zero integer.

Example : (a) $\frac{-7}{8} = \frac{-7 \times 2}{8 \times 2} = \frac{-14}{16}$ (b) $\frac{13}{18} = \frac{13 \times 3}{18 \times 3} = \frac{39}{54}$ (c) $\frac{3}{-13} = \frac{3 \times (-3)}{-13 \times (-3)} = \frac{-9}{39}$

$\frac{-7}{8} = \frac{-7 \times (-5)}{8 \times (-5)} = \frac{35}{-40}$ $\frac{13}{18} = \frac{13 \times (-2)}{18 \times (-2)} = \frac{-26}{-36} = \frac{26}{36}$ $\frac{3}{-13} = \frac{3 \times 6}{-13 \times 6} = \frac{18}{-78}$

Example 5 : Show that rational numbers $\frac{3}{5}$ and $\frac{18}{30}$ are equivalent.

Solution : We know that for two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ to be equivalent ad = bc.

Let $\frac{a}{b} = \frac{3}{5}$ and $\frac{c}{d} = \frac{18}{30}$

ad = 3 × 30 = 90 and bc = 5 × 18 = 90

Since ad = bc = 90, So $\frac{3}{5}$ and $\frac{18}{30}$ are equivalent.

Example 6 : Write four equivalent rational numbers of $\frac{6}{11}$.

Solution : $\frac{6}{11} = \frac{6 \times 2}{11 \times 2} = \frac{12}{22}$, $\frac{6}{11} = \frac{6 \times 3}{11 \times 3} = \frac{18}{33}$

$\frac{6}{11} = \frac{6 \times 4}{11 \times 4} = \frac{24}{44}$, $\frac{6}{11} = \frac{6 \times 5}{11 \times 5} = \frac{30}{55}$

So, four equivalent rational numbers of $\frac{6}{11}$ are $\frac{12}{22}, \frac{18}{33}, \frac{24}{44}$ and $\frac{30}{55}$.

Example 7 : Express $\frac{-17}{15}$ as a rational number with numerator 85.

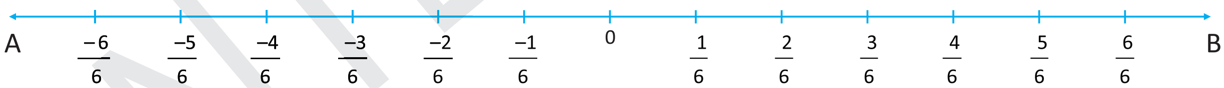
Solution : To express $\frac{-17}{15}$ as a rational number with numerator 85 which when multiplied by (–17), gives 85.

Since 85 ÷ 17 = 5, so multiply both the numerator and denominator by (–5) as (–17) × (–5) = 85.

$\frac{-17}{15} = \frac{(-17) \times (-5)}{15 \times (-5)} = \frac{85}{-75}$ So, $\frac{-17}{15} = \frac{85}{-75}$

Comparison of Rational Numbers

Have a look at the number line representing rational numbers as shown in the figure.



From the figure, we learn that

$\frac{-6}{6} < \frac{-5}{6} < \frac{-4}{6} < \frac{-3}{6} < \frac{-2}{6} < \frac{-1}{6} < 0 < \frac{1}{6} < \frac{2}{6} < \frac{3}{6} < \frac{4}{6} < \frac{5}{6} < \frac{6}{6}$ (ascending order)

and $\frac{6}{6} > \frac{5}{6} > \frac{4}{6} > \frac{3}{6} > \frac{2}{6} > \frac{1}{6} > 0 > \frac{-1}{6} > \frac{-2}{6} > \frac{-3}{6} > \frac{-4}{6} > \frac{-5}{6} > \frac{-6}{6}$ (descending order)

The above mentioned rational numbers have same denominator. Therefore, by comparing the numerators we find out which is greater or smaller.

Example : $\frac{-4}{6} > \frac{-5}{6}$ (Since $-4 > -5$)

$\frac{-2}{6} < \frac{5}{6}$ (Since $-2 < 5$)

Other than this, all other rules for the comparison of integers and fractions are applicable to rational numbers also.

(i) All positive rational numbers are greater than 0.

(ii) All positive rational numbers are greater than all negative rational numbers.

(ii) All negative numbers are smaller than 0.

Example : $\frac{-3}{17} < \frac{-1}{17}$ [Since $(-3) < (-1)$]

and $\frac{3}{7} < \frac{18}{7}$ [Since $3 < 18$]

When the rational numbers have different denominators we change them as rational numbers with the same denominator and than compare. We will learn it through examples.

Example 8 : Arrange the following in descending order :

(a) $\frac{1}{12}, \frac{3}{12}, \frac{6}{12}, \frac{5}{12}, \frac{13}{12}$ (b) $\frac{-4}{13}, \frac{4}{13}, \frac{-1}{13}, \frac{12}{13}$

Solution : (a) Since the denominator of the given rational numbers is same i.e. 12, we write their numerators in descending order.

Here, $13 > 6 > 5 > 3 > 1$

So, $\frac{13}{12} > \frac{6}{12} > \frac{5}{12} > \frac{3}{12} > \frac{1}{12}$

(b) Since the denominator of the given rational numbers is same, i.e. 13, so, we write their numerators in descending order:

Here, $12 > 4 > -1 > -4$

So, $\frac{12}{13} > \frac{4}{13} > \frac{-1}{13} > \frac{-4}{13}$

Example 9 : Which is greater : $\frac{5}{6}$ or $\frac{7}{9}$?

Solution : Since the denominators of the given rational numbers are different, we change them as rational numbers with the same denominator and then compare.

$\frac{5}{6} = \frac{5 \times 3}{6 \times 3} = \frac{15}{18}, \frac{7}{9} = \frac{7 \times 2}{9 \times 2} = \frac{14}{18}$

$\frac{15}{18}$ $\boxed{?}$ $\frac{14}{18}$

Since $15 > 14$ So, $\frac{15}{18} > \frac{14}{18}$ or, $\frac{5}{6} > \frac{7}{9}$

Facts to Know

- We can also use the following relations to compare two rational numbers :

(i) If $\frac{a}{b} > \frac{c}{d} = ad > bc$ (ii) If $\frac{a}{b} < \frac{c}{d} = ad < bc$

Exercise 2.1

1. **Which of the following is not a rational number ?**

   $\frac{-2}{7}, \frac{4}{-13}, 1, \frac{2}{9}, 0, \frac{0}{7}, \frac{6}{0}, \frac{7}{5}$

2. **Convert the following rational numbers into integers:**

   $\frac{-4}{-1}, \frac{-42}{14}, \frac{-36}{-18}, \frac{7}{-1}$

3. **Identify the positive rational numbers from the following :**

   (a) $\frac{-3}{-4}$ (b) $\frac{6}{8}$ (c) $\frac{-8}{7}$ (d) $\frac{9}{-14}$ (e) $\frac{-8}{-14}$ (f) $\frac{-1}{7}$ (g) $\frac{2}{-6}$ (h) $\frac{-277}{643}$

4. Write any five positive and five negative rational numbers.

5. **Represent the following on a number line:**

   (a) $\frac{-3}{4}$ (b) $\frac{-5}{3}$ (c) $\frac{3}{5}, \frac{1}{5}, \frac{7}{5}, \frac{4}{5}$ (d) $\frac{-7}{8}, \frac{-5}{8}, \frac{-3}{8}, \frac{1}{8}, \frac{3}{8}$

6. **Express the following in standard form :**

   (a) $\frac{85}{280}$ (b) $\frac{27}{243}$ (c) $\frac{-12}{156}$ (d) $\frac{70}{357}$

7. **Write four equivalent rational numbers for the following:**

   (a) $\frac{-3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{-11}{7}$ (d) $\frac{6}{11}$

8. **Find the value of *x* in the following:**

   (a) $\frac{x}{-9} = \frac{15}{12}$ (b) $\frac{-3}{4} = \frac{18}{x}$ (c) $\frac{4}{7} = \frac{-12}{x}$ (d) $\frac{-2}{5} = \frac{-8}{x}$ (e) $\frac{27}{15} = \frac{x}{24}$

9. **Observe the given patterns carefully and write the next two rational numbers for the following :**

   (a) $\frac{7}{11}, \frac{5}{13}, \frac{3}{15}, \frac{1}{17}, \frac{-1}{19}$, ___, ___. (b) $\frac{5}{7}, \frac{7}{14}, \frac{9}{21}, \frac{11}{28}$, ___, ___.

   (c) $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, ___, ___. (d) $\frac{-3}{7}, \frac{-4}{7}, \frac{-5}{7}, \frac{-6}{7}$, ___, ___.

10. **Arrange the following rational numbers in ascending order :**

    (a) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{5}, \frac{1}{6}$ (b) $\frac{-2}{7}, \frac{4}{4}, \frac{6}{7}, \frac{-3}{7}, \frac{-4}{7}$

11. **Arrange the following rational numbers in descending order:**

    (a) $\frac{-2}{3}, \frac{4}{3}, \frac{2}{3}, \frac{-1}{3}, \frac{1}{3}, \frac{7}{3}, \frac{-7}{3}$ (b) $\frac{1}{3}, \frac{6}{3}, \frac{7}{3}, \frac{-13}{3}, \frac{-14}{3}, \frac{2}{3}$ (c) $\frac{-1}{2}, \frac{-3}{4}, \frac{4}{3}, \frac{2}{6}, \frac{2}{5}$ (d) $\frac{-7}{5}, \frac{-2}{5}, \frac{2}{5}, \frac{7}{10}, \frac{-6}{5}$

12. **Which of the two given rational numbers is smaller ?**

    (a) $\frac{-3}{7}, \frac{-4}{9}$ (b) $\frac{14}{27}, \frac{7}{9}$ (c) $\frac{-6}{17}, \frac{-8}{15}$ (d) $\frac{-4}{7}, \frac{-3}{4}$

Addition of Rational Numbers

For addition of rational numbers, we should follow these steps:

(i) Express each rational number with positive denominator.

(ii) If the denominators are same, add their numerator and divide it by common denominator.

(iii) If the denominators are different, express them as equivalent rational numbers with same denominator. Now add the numerators and divide it by the common denominator of the equivalent rational numbers .

(iv) If required, change the result into standard form.

Example 10 : **Add the following:**

(a) $\frac{-8}{19}+\frac{-2}{57}$ (b) $\frac{-18}{29}+\left(\frac{-4}{29}\right)$

Solution : (a) $\frac{-8}{19}+\frac{-2}{57}=\frac{-8\times 3+(-2)\times 1}{57}$

$=\frac{-24+(-2)}{57}=\frac{-26}{57}$ [LCM of 19 and 57 in 57]

(b) $\frac{-18}{29}+\frac{-4}{29}=\frac{-18+(-4)}{29}$

$=\frac{-18-4}{29}=\frac{-22}{29}$

Example 11 : **Add the following:**

(a) $\frac{-17}{4}$ and $\frac{-37}{8}$ (b) $\frac{-4}{9}$ and $\frac{-7}{27}$

Solutions : (a) $\frac{-17}{4}+\left(\frac{-37}{8}\right)$

$=\frac{-17\times 2}{4\times 2}+\left(\frac{-37}{8}\right)$ (8 is a multiple of 4)

$=\frac{-34}{8}+\left(\frac{-37}{8}\right)$

$=\frac{-34-37}{8}=\frac{-71}{8}$

(b) $\frac{-4}{9}+\left(\frac{-7}{27}\right)$

$=\frac{-4\times 3}{9\times 3}+\left(\frac{-7}{27}\right)$ (27 is a multiple of 9)

$=\frac{-12}{27}+\left(\frac{-7}{27}\right)$

$=\frac{-12-7}{27}=\frac{-19}{27}$

Example 12 : **Solve the following:**

(a) $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}$ (b) $\frac{-3}{4}+\frac{7}{10}+\left(\frac{-5}{12}\right)$

Solution : (a) $\frac{2}{3}+\frac{4}{5}+\frac{3}{4}=\frac{2\times 20}{3\times 20}+\frac{4\times 12}{5\times 12}+\frac{3\times 15}{4\times 15}$

[LCM of 3, 5 and 4 = 60]

$=\frac{40}{60}+\frac{48}{60}+\frac{45}{60}$

$=\frac{40+48+45}{60}=\frac{133}{60}$

(b) $\frac{-3}{4}=\frac{-3\times 15}{4\times 15}=\frac{-45}{60}$

$\frac{7}{10}=\frac{7\times 6}{10\times 6}=\frac{42}{60}$

$\frac{-5}{12}=\frac{-5\times 5}{12\times 5}=\frac{-25}{60}$

[LCM of 4, 10 and 12 = 60]

Now, $\frac{-3}{4}+\frac{7}{10}+\left(\frac{-5}{12}\right)=\frac{-45}{60}+\frac{42}{60}+\left(\frac{-25}{60}\right)$

$=\frac{-45+42-25}{60}=\frac{-28}{60}=\frac{-7}{15}$

Subtraction of Rational Numbers

We know that subtraction is the inverse process of addition. We can subtract a rational numbers from another by adding its additive inverse to it. Let $\frac{p}{q}$ and $\frac{r}{s}$ be two rational numbers. To subtract $\frac{r}{s}$ from $\frac{p}{q}$ we add additive inverse of $\frac{r}{s}$ to $\frac{p}{q}$.

Thus, $\frac{p}{q}-\frac{r}{s}=\frac{p}{q}+\left(\frac{-r}{s}\right)$

Example 13 : Subtract $\frac{-7}{8}$ from $\frac{5}{7}$.

Solution : The additive inverse of $\frac{-7}{8}=-\left(\frac{-7}{8}\right)=\frac{7}{8}$

Now, we have to add additive inverse of $\frac{-7}{8}$ to $\frac{5}{7}$

$$\frac{5}{7}-\left(\frac{-7}{8}\right)=\frac{5}{7}+\frac{7}{8}$$

$$=\frac{(5\times8)+(7\times7)}{56}$$

$$=\frac{40+49}{56}$$

$$=\frac{89}{56}$$

Example 14 : Subtract $\frac{5}{7}$ from $\frac{18}{7}$.

Solutions : $\frac{18}{7}-\frac{5}{7}$

$$\frac{18}{7}-\frac{5}{7}=\frac{18}{7}+\left(\frac{-5}{7}\right)$$

$$=\frac{18+(-5)}{7}$$

$$=\frac{13}{7}$$

Facts to Know

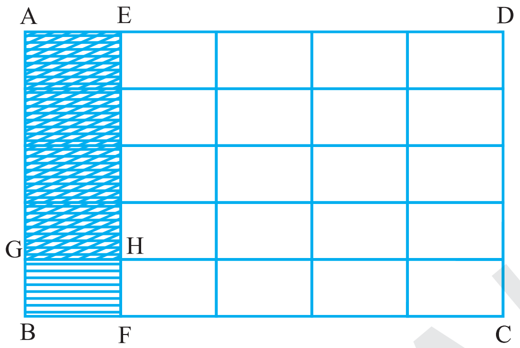
- There are infinite rational numbers between two consecutive rational numbers.

Multiplication of Rational Numbers

Let's learn multiplication of rational numbers $\left(\frac{4}{5}\times\frac{1}{5}\right)$ with the help of a diagram.

The shaded part ABFE shows $\frac{1}{5}$ of ABCD

The double-shaded part AGHE shows $\frac{4}{5}$ out of $\frac{1}{5}$.



From the given figure, it is clear that the double–shaded part shows $\frac{4}{25}$

So, $\frac{4}{5}$ of $\frac{1}{5}=\frac{4}{25}$

or $\frac{4}{5}\times\frac{1}{5}=\frac{4}{25}$

Also, we observe that $\frac{4}{5}\times\frac{1}{5}=\frac{4\times1}{5\times5}$

Hence, it can be concluded that the product of two rational numbers is a rational number whose numerator is the product of numerators of the denominators. If $\frac{p}{q},\frac{r}{s}$ and $\frac{t}{u}$ are three rational numbers, then

$$\frac{p}{q}\times\frac{r}{s}\times\frac{t}{u}=\frac{p\times q\times t}{q\times s\times u}=\frac{\text{Product of numerators}}{\text{Product of denominators}}$$

Example 15 : Find the product of $\frac{-12}{17}$ and $\frac{8}{5}$.

Solution : $\frac{-12}{17}\times\frac{8}{5}=\frac{(-12)\times8}{17\times5}$

$=\frac{-96}{85}$

Example 16 : Find the product of $\frac{2}{3},\frac{-4}{5},\frac{6}{7}$ and $\frac{-3}{4}$.

Solution : $\frac{2}{3}\times\left(\frac{-4}{5}\right)\times\frac{6}{7}\times\left(\frac{-3}{4}\right)=\frac{2\times(-4)\times6\times(-3)}{3\times5\times7\times4}=\frac{12}{35}$

Division of Rational Numbers

It is known to us that division is the inverse of multiplication i.e., if p and q two integers, then p ÷ q = $p\times\frac{1}{q}$, Here we multiply the dividend by the multiplicative inverse of the divisor. The same rule is applied for division of rational numbers. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}$ $\left[\frac{c}{d}\neq0\right]$

Example 17 : Divide $\frac{-8}{9}$ by $\frac{12}{13}$.

Solution : $\frac{-8}{9}\div\frac{12}{13}=\frac{-8}{9}\times\frac{13}{12}$ $\left[\textit{Multiplicative inverse of } \frac{12}{13} \textit{ is } \frac{13}{12}\right]$

$=\frac{-8\times13}{9\times12}=\frac{-26}{27}$

Facts to Know

- The sum of a rational number and its additive inverse is 0 (zero).
- The product of a rational number and its reciprocal is always 1.

Example 18 : Divide $\frac{-3}{7}$ by $\frac{3}{5}$.

Solution : $\frac{-3}{7} \div \frac{3}{5} = \frac{-3}{7} \times \frac{5}{3}$ [*Multiplicative inverse of* $3/5$ *is* $5/3$]

$$= \frac{(-3) \times 5}{7 \times 3}$$

$$= \frac{-5}{7}$$

Exercise 2.2

1. Solve the following:

(a) $\frac{3}{5}+\frac{6}{5}+\frac{8}{5}$ (b) $\frac{-5}{6}+\left(\frac{-3}{7}\right)$ (c) $\frac{4}{7}+\left(\frac{-2}{6}\right)+\left(\frac{-2}{3}\right)$

(d) $\frac{-3}{11}+\left(\frac{-4}{11}\right)$ (e) $\frac{5}{7}+\frac{4}{21}+\left(\frac{-3}{14}\right)$ (f) $\frac{-3}{4}+\left(\frac{-4}{5}\right)+\left(\frac{-6}{7}\right)+\left(\frac{-7}{8}\right)$

2. Add the following:

(a) $\frac{3}{4}, \frac{5}{6}$ and $\frac{-2}{3}$ (b) $\frac{-8}{7}$ and $\frac{-4}{7}$ (c) $\frac{13}{15}$ and $\frac{-7}{25}$

(d) $\frac{1}{3}$ and $\frac{-3}{4}$ (e) $\frac{5}{6}$ and $\frac{-6}{7}$ (f) $\frac{-7}{15}$ and $\frac{12}{5}$

3. Subtract the following :

(a) 14 from $\frac{-7}{8}$ (b) $\frac{-4}{5}$ from $\frac{-7}{12}$ (c) $\frac{4}{5}$ from $\frac{-4}{7}$

(d) $\frac{5}{7}$ from $\frac{7}{5}$ (e) $\frac{-3}{8}$ from $\frac{-7}{8}$ (f) $\frac{-5}{8}$ from $\frac{-11}{12}$

4. Find the product of the following:

(a) $\frac{4}{5}\times\left(\frac{-15}{21}\right)\times\frac{14}{25}$ (b) $\frac{-7}{8}\times\left(\frac{-5}{6}\right)$ (c) $\frac{-3}{4}\times\left(\frac{-5}{6}\right)\times\left(\frac{-7}{8}\right)$

(d) $\frac{-5}{7}\times\left(\frac{-8}{15}\right)\times\left(\frac{21}{16}\right)$ (e) $\frac{4}{7}\times\left(\frac{-3}{5}\right)\times\frac{14}{24}$ (f) $\frac{-11}{5}\times\left(\frac{-15}{22}\right)\times\frac{3}{5}$

5. Simplify the following:

(a) $\left\{\frac{4}{7}\times\frac{21}{16}\right\}\div\frac{7}{8}$ (b) $\frac{-5}{8}+\frac{6}{7}-\frac{2}{3}$ (c) $\frac{-3}{4}\times\left[\frac{3}{5}-\frac{2}{3}\right]$

(d) $\frac{-7}{4}\times\left[\frac{-7}{8}+\frac{9}{16}\right]$ (e) $\frac{-3}{5}-\frac{(-4)}{15}+\frac{-7}{10}$

6. The sum of two rational numbers is $\frac{-3}{7}$. If one of them is $\frac{2}{3}$, find the other number.

7. What should be added to $\left(\frac{-3}{4}+\frac{5}{7}\right)$ to get $\frac{-6}{14}$.

8. Find the product of $\frac{-3}{8}$ and the reciprocal of $\frac{15}{16}$.

Rational Numbers as Decimals

We can express a rational number into a decimal either by the long division method or by writing an equivalent rational number with the denominator as power of 10, and then write the equivalent rational number as a decimal.

Example 19 : **Express $\frac{7}{4}$ as a decimal number by writing an equivalent rational number with denominator as power of 10.**

Solution : $\frac{7}{4} = \frac{7 \times 25}{4 \times 25} = \frac{175}{100} = 1.75$

$\therefore \frac{7}{4} = 1.75$

Example 20 : **Express $\frac{3}{25}$ as a decimal number by using long division method.**

Solution : $\frac{3}{25} = 3 \div 25$

$$\begin{array}{r} 25\overline{)3.0}(0.12 \\ -25 \\ \hline 50 \\ -50 \\ \hline 0 \\ \hline \end{array}$$

$\therefore \frac{3}{25} = 0.12$

Terminating and Non-Terminating Decimals

When a rational number is converted into decimals by division method, any one of the following two conditions will arise:

(a) The division process comes to an end after some steps, as there is no remainder left at certain point of time. Such decimals are called terminating decimals.

Example : $\frac{1}{2} = 0.5, \quad \frac{-3}{8} = -0.375$ etc.

(b) The division process goes on indefinitely as there may be a remainder at each step. Such decimals are called non-terminating decimals.

Example : $\frac{1}{3} = 0.333....$

$$\begin{array}{r} 0.333.... \\ 3\overline{)\,10} \\ -9 \\ \hline 10 \\ -9 \\ \hline 10 \\ -9 \\ \hline 1 \\ \hline \end{array}$$

The rational number in which the division process does not come to an end and keeps repeating, is called non-terminating repeating decimal. To represent such decimals, we put a bar sign (–) above the repeating part. So, $\frac{1}{3} = 0.333... = 0.\overline{3}$.

Example 21 : **Express the following rational numbers as decimals using the division method. Do write whether these represent terminating or non-terminating decimals.**

(a) $\frac{5}{6}$ (b) $\frac{2}{7}$ (c) $\frac{5}{8}$

Solution : (a) $\frac{5}{6} = 5 \div 6$

6) 50000 (0.8333
−48
20
−18
20
−18
20
−18
2

$\therefore \frac{5}{6} = 0.8333 ...... = 0.8\overline{3}$

Thus, $\frac{5}{6} = 0.8\overline{3}$ represent non-terminating decimals.

(b) $\frac{2}{7} = 2 \div 7$

7) 2.00000000 (0.28571428
−14
60
−56
40
−35
50
−49
10
−7
30
−28
20
−14
60
−56
4

$\therefore \frac{2}{7} = 0.28571428.... = 0.\overline{285714}$

Thus $\frac{2}{7} = 0.\overline{285714}$ represents non-terminating decimals.

(c) $\frac{5}{8} = 5 \div 8$

8) 5.000 (0.625
−48
20
−16
40
−40
0

$\therefore \frac{5}{8} = 0.625$

Thus, $\frac{5}{8} = 0.625$ represents terminating decimals.

It is to note that non-terminating non-repeating decimals can not be converted into rational number. Such type of numbers are called irrational numbers.

Facts to Know

- Every rational number can be converted into either a terminating decimal or non-terminating repeating decimal.
- Such decimals which are non-terminating and have no repeating parts are called irrational numbers.

Rule to Find Terminating or Non-terminating Repeating Decimals

Rule for Terminating Decimals : If a rational number is in its lowest term and its denominator has no multiple other than 2 or 5 or both.

Example : $\frac{1}{4}, \frac{3}{25}$ and $\frac{1}{250}$ represent terminating decimals.

Rule for Non-terminating Repeating Decimals : If a rational number is in its lowest term and its denominator has a prime factor other than 2 and 5.

Example : $\frac{3}{7}, \frac{5}{6}$ and $\frac{7}{15}$ are examples of non-terminating repeating decimals.

Example 22 : **Without actual division, determine which of the following rational numbers have a terminating decimal representation:**

(a) $\frac{7}{225}$ (b) $\frac{3}{16}$ (c) $\frac{9}{75}$

Solution : (a) In $\frac{7}{225}$ the denominator is 225.

We have, $225 = 5 \times 5 \times 3 \times 3$

Thus, 225 has 3 as a prime number that is other than 2 and 5.

Hence, $\frac{7}{225}$ must have a non-terminating decimal representation.

(b) In $\frac{3}{16}$ the denominator is 16.

We have, $16 = 2 \times 2 \times 2 \times 2$

Thus, 16 has 2 as the only prime factor.

Hence, $\frac{3}{16}$ must have a terminating decimal representation.

(c) In $\frac{9}{75}$ the denominator is 75.

We have, $75 = 5 \times 5 \times 3$

Thus, 75 has 3 as a prime number that is other than 2 and 5.

Hence, $\frac{9}{75}$ must have a non-terminating decimal representation.

Conversion of Non-terminating Repeating Decimals into Rational Numbers

There are two types of decimal representation of non-terminating repeating decimals:

(i) **Pure Repeating or Recurring Decimal :** A decimal presentation in which all the digits after the decimal point are repeated.

Example : 0.67, 0.7 and 0.123 are recurring decimals.

(ii) **Mixed Repeating or Recurring Decimal :** A decimal presentation in which at least one digit after the decimal point is non-repeating.

Example : $1.2\overline{345}$, $4.2\overline{35}$ and $1.01\overline{25}$ are mixed recurring decimals.

Let's learn conversion of non-terminating repeating decimals into rational numbers through examples.

Example 23 : **Convert the following decimals in the form of $\frac{p}{q}$:**

(a) $0.\overline{8}$ (b) $0.\overline{87}$ c) $7.\overline{23}$ (d) $0.72\overline{3}$

Solution : (a) Let $x = 0.\overline{8}$ ....(i)

Here, only one digit in decimal part is repeated, we multiply it by 10, we get,

$10x = 8.\overline{8}$ .... (ii)

Subtracting (i) from (ii), we get,

$10x - x = 8.\overline{8} - 0.\overline{8}$

$\Rightarrow 9x = 8$

$\Rightarrow x = \frac{8}{9}$

(b) Let $x = 0.\overline{87}$ ......(i)

Here, only two digits in decimal part is repeated, we multiply it by 100, we get,

$100x = 87.\overline{87}$ ....(ii)

Subtracting (i) from (ii), we get,

$\Rightarrow \quad 99x = 87$

$\Rightarrow \quad x = \frac{87}{99}$

(c) Let $x = 7.\overline{23}$ ....(i)

Here, only two digits in decimal part is repeated, we multiply it by 100, we get,

$100x = 723.\overline{23}$ ....(ii)

Subtracting (i) from (ii) we get,

$100x - x = 723.\overline{23} - 7.\overline{23}$

$\Rightarrow \quad 99x = 716$

$\Rightarrow \quad x = \frac{716}{99}$

(d) Let $x = 0.72\overline{3}$

Here, we have 3 digits in the decimal part, out of which only one is repeating.

First we multiply it by 100 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore \quad 100x = 72.\overline{3}$ .... (i)

Now, only one digit is repeating, so we again multiply it by 10, we get

$1000x = 723.\overline{3}$ .... (ii)

Subtracting (i) from (ii), we get,

$1000 - 100x = 723.\overline{3} - 72.\overline{3}$

$\Rightarrow \quad 900x = 651$

$\Rightarrow \quad x = \frac{651}{900}$

Short-cut Method of Converting a Non-terminating Decimal into Rational Numbers:

To convert a recurring decimals into $\frac{p}{q}$ form, write repeated figure only once in the numerator and take as many nines in the denominator as the number of repeated digits

Example : $0.\overline{37} = \frac{37}{99}$ and $0.\overline{123} = \frac{123}{999}$

To convert a mixed recurring decimal into $\frac{p}{q}$ form, its numerator is obtained by removing the decimal point and bar and then subtract the non-repeating number. The denominator will carry as many nines as the number of digits in the repeating part followed by as many zero as the numbers of digits in the non-repeating part after decimal point.

Example : $0.2\overline{37} = \frac{237-2}{990} = \frac{235}{990}$

Example 24 : **Evaluate the following using short-cut method:**

(a) $3.\overline{67} + 4.\overline{58}$ (b) $2.3\overline{53} - 1.1\overline{25}$

Solution : (a) $3.\overline{67} + 4.\overline{58} = (3+4) + (0.\overline{67} + 0.\overline{58})$

$= 7 + \left(\frac{67}{99} + \frac{58}{99}\right)$

$$= 7 + \frac{125}{99}$$

$$= 7 + 1\frac{26}{99}$$

$$= 8\frac{26}{99}$$

(b) $2.3\overline{53} - 1.1\overline{25}$

$$= (2 + 0.3\overline{53}) - (1 + 0.1\overline{25})$$

$$= \left(2 + \frac{353-3}{990}\right) - \left(1 + \frac{125-1}{990}\right)$$

$$= 2 + \frac{350}{990} - 1 - \frac{124}{990}$$

$$= (2-1) + \left(\frac{350}{990} - \frac{124}{990}\right)$$

$$= 1 + \frac{350-124}{990}$$

$$= 1 + \frac{226}{990}$$

$$= 1\frac{113}{495}$$

Exercise 2.3

1. Convert the following into decimals by writing an equivalent rational number with denominator as power of 10:

(a) $\frac{13}{5}$ (b) $\frac{-7}{25}$ (c) $\frac{3}{125}$ (d) $\frac{-7}{40}$

2. Without actual division state whether the following rational numbers represent terminating or non-terminating decimals :

(a) $\frac{5}{18}$ (b) $\frac{17}{8}$ (c) $\frac{-23}{75}$ (d) $\frac{27}{64}$

(e) $\frac{-7}{25}$ (f) $\frac{12}{29}$ (g) $\frac{8}{125}$ (h) $\frac{-19}{20}$

3. Convert the following rational numbers into decimal numbers:

(a) $\frac{2}{15}$ (b) $\frac{3}{7}$ (c) $\frac{7}{8}$ (d) $\frac{17}{25}$

(e) $\frac{12}{24}$ (f) $\frac{13}{4}$ (g) $\frac{125}{8}$ (h) $\frac{12}{25}$

4. Convert the following decimals into rational numbers:

(a) 1.25 (b) 7.025 (c) $3.\overline{7}$ (d) $2.3\overline{45}$

(e) $0.2\overline{3}$ (f) $3.\overline{356}$ (g) $0.7\overline{8}$ (h) $1.02\overline{3}$

5. **Find the value of the following :**

(a) $2.\overline{3}+3.\overline{4}$ (b) $3.\overline{34}+6.\overline{78}$ (c) $1.2\overline{35}-0.7\overline{85}$ (d) $18.\overline{63}-7.6\overline{83}$

6. If $\frac{x}{y} = 2.3\overline{6} + 1.7\overline{3}$, find $\frac{x}{y}$.

7. If $\frac{x}{y} = 1.\overline{356} - 1.\overline{067}$, find the least value of x and y.

8. **Which of the following decimals can be expressed as rational numbers ?**

(a) 0.3333 .... (b) 0.127272727............. (c) 3.4010010001.............
(d) 13.35735735 .... (e) 2.0010020003............. (f) 7.125125.............

Points to Remember

- A rational number can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
- All fractions are rational numbers but all rational numbers are not fractions.
- All counting numbers, whole numbers and integers are rational numbers.
- All rational numbers can be represented on a numbers line.
- The absolute value of a rational number is equal to its positive numeric value.
- For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$:

  (i) $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$ (Equivalent) (ii) $\frac{a}{b} > \frac{c}{d} \Rightarrow ad > bc$ (iii) $\frac{a}{b} < \frac{c}{d} \Rightarrow ad < bc$
- A rational number is in standard form if the HCF of its numerator and denominator is 1.
- For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a}{b} + \frac{c}{d} = 0$ then $\frac{c}{d}$ is called the additive inverse of $\frac{a}{b}$ and vice versa.
- The product of a rational number and its reciprocal is always 1.
- A rational number can be converted into decimal either by long division method or by writing an equivalent rational number with the denominator as power of 10.
- Rational numbers can be expressed either as terminating or non-terminating repeating decimals.
- Decimals which are non-terminating and non-repeating are called irrational numbers.

EXERCISE

1. **MULTIPLE CHOICE QUESTIONS (MCQs):**

Tick (✓) the correct options :

(a) Only rational number whose absolute value is 0, is :

(i) 0 ☐ (ii) −1 ☐ (iii) +1 ☐ (iv) −9 ☐

(b) Additive inverse of $\frac{-17}{14}$ is :

(i) $\frac{14}{17}$ ☐ (ii) $\frac{17}{14}$ ☐ (iii) $\frac{17}{-14}$ ☐ (iv) $\frac{-14}{-17}$ ☐

(c) Only rational number which is neither positive nor negative is :

(i) 0 ☐ (ii) 1 ☐ (iii) 100 ☐ (iv) None of these ☐

(d) The largest rational numbers is :

(i) 2^n ☐ (ii) 10^n ☐ (iii) $\left(\frac{p}{q}\right)^n$ ☐ (iv) non determinable ☐

(e) Multiplicative inverse of $\frac{-7}{18}$ is :

(i) $\frac{18}{7}$ (ii) $\frac{18}{-7}$ (iii) $\frac{7}{18}$ (iv) $\frac{-7}{-18}$

(f) $\frac{-312}{169}$ in the standard form is :

(i) $\frac{24}{14}$ (ii) $\frac{27}{13}$ (iii) $\frac{24}{13}$ (iv) $\frac{29}{17}$

(g) If $\frac{-x}{6}=\frac{42}{-36}$, then the value of x is:

(i) 7 (ii) −7 (iii) 14 (iv) −14

(h) The example of irrational number is :

(i) $\frac{2}{5}$ (ii) $\frac{3}{7}$ (iii) $\frac{7}{100}$ (iv) 0.1234564325624....

2. Represent the following on the numbers line:

(a) $\frac{-5}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{-2}{5}$ and $\frac{2}{5}$ (d) $\frac{-3}{5}$ and $\frac{3}{5}$

3. Convert the following rational numbers in standard form:

(a) $\frac{-65}{225}$ (b) $\frac{161}{343}$ (c) $\frac{-125}{1000}$ (d) $\frac{55}{3250}$

4. Find the absolute value of the following rational numbers:

(a) $\frac{-3}{4}+\frac{3}{7}-\frac{6}{8}$ (b) $\frac{5}{9}+\left(\frac{-6}{24}\right)$ (c) $\frac{-2}{3}-\frac{3}{5}$ (d) $3-\frac{4}{5}$

5. In each of the following pairs of rational numbers, which is greater ?

(a) $\frac{3}{4},\frac{5}{7}$ (b) $\frac{-7}{3},-\frac{5}{3}$ (c) $\frac{5}{11}$ and $\frac{-3}{-7}$ (d) $\frac{-2}{3},\frac{3}{4}$

6. Arrange the following rational numbers in ascending order:

(a) $\frac{2}{3},\frac{3}{4},\frac{5}{8},\frac{-7}{8},\frac{-5}{6}$ (b) $\frac{-5}{14},\frac{3}{10},\frac{-3}{7},\frac{-6}{35}$ (c) $\frac{-2}{3},\frac{-5}{6},\frac{2}{3},\frac{5}{6},\frac{-1}{3},\frac{1}{3}$ (d) $-2,\frac{-1}{2},\frac{1}{2},0,2,\frac{-3}{5},\frac{3}{5}$

7. Simplify :

(a) $\frac{-3}{7}+\left(\frac{-4}{7}\right)+\left(\frac{-6}{7}\right)$ (b) $\frac{3}{5}+\left(\frac{-4}{14}\right)+\left(\frac{-2}{5}\right)+\frac{6}{15}$ (c) $\frac{-6}{17}+\frac{5}{51}$ (d) $\frac{-1}{2}-\frac{2}{3}-\frac{3}{4}-\frac{5}{6}$

8. Solve :

(a) $\frac{-7}{17}-0$ (b) $\frac{-12}{13}-\left(\frac{-15}{26}\right)$ (c) $\frac{-8}{26}-\left(\frac{24}{26}\right)$ (d) $\frac{9}{13}-\left(\frac{-4}{13}\right)$

9. Simplify: (a) $\frac{-3}{8}\times\frac{16}{15}\times\left(\frac{-24}{75}\right)\times\frac{20}{12}$ (b) $\frac{-6}{5}\times\left(\frac{-4}{7}\right)\times\left(\frac{21}{-12}\right)\times\left(\frac{-3}{-4}\right)$

10. Divide the sum of $\frac{7}{8}$ and $\frac{-3}{4}$ by the product of $\frac{-2}{3}$ and $\frac{-3}{4}$.

11. What rational number should we multiply to $\frac{-5}{6}$ to get the product 24 ?

12. If $x=\frac{-3}{4}$ and $y=\frac{-2}{3}$, find the value of $3x-4y$.

13. Find the value of *x* for the following :

(a) $\frac{-6}{7} + x = \frac{-7}{8}$ (b) $\frac{2}{5} \div x = \frac{-3}{7}$ (c) $x - \left(\frac{-5}{7}\right) = \frac{7}{5}$

14. Convert the following rational numbers into decimals:

(a) $\frac{2}{5}$ (b) $\frac{6}{7}$ (c) $\frac{3}{50}$ (d) $\frac{5}{12}$

15. Convert the following decimals into rational numbers:

(a) 0.025 (b) 3.15 (c) $25.\overline{5}$ (d) $0.3\overline{79}$

16. Simplify of the following:

(a) $4.2\overline{3} + 3.\overline{79}$ (b) $2.\overline{37} - 3.\overline{25} + 1.\overline{23}$ (c) $0.\overline{78} + 0.6\overline{7}$ (d) $3.7\overline{86} + 37.\overline{86}$

17. If $\frac{p}{q} = 3.\overline{7} + 7.\overline{3}$, find $\frac{p}{q}$.

18. Evaluate the following:

(a) $2.\overline{3} \times 1.\overline{2}$ (b) $1.2\overline{5} \div 0.\overline{5}$

HOTS

There are 100 students in a school. Each student is required to participate in an extracurricular activity. The choices are art, cricket, basketball and swimming. $\frac{3}{10}$ of students are in art, $\frac{1}{10}$ are in cricket and 17 play basketball. How many students are going to participate in swimming?

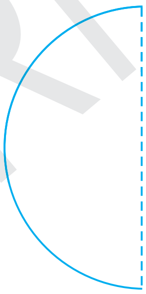
Lab Activity

Objective : To multiply two rational numbers by folding circular paper.

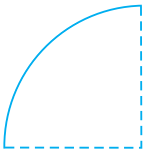
Materials Required : Circular paper strip and sketch pen.

Procedure : Let us find the product of two rational numbers, say $\frac{2}{4}$ and $\frac{1}{2}$. Take circular paper strip and follow these steps :

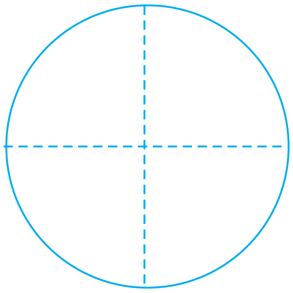
Step 1 : Fold the circular strip into two halves



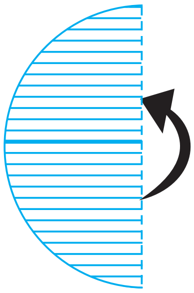
Step 2 : Fold the strip again.



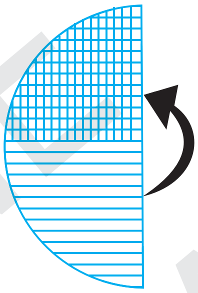
Step 3 : Unfold the strip and your will get the shape like this.



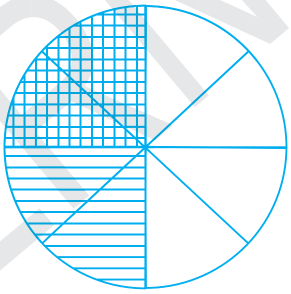
Step 4 : Shade two parts with horizontal lines and fold the rest unshaded part.



Step 5 : Now, fold the strip horizontally to divide the strip into two parts and shade it with vertical lines in one out of two parts.



Step 6 : Unfold the strip. You will find that 2 out of 8 parts are double shaded. This illustrates that the double shaded region represents $\frac{2}{8}$ of the whole strip.



Conclusion : $\frac{2}{4} \times \frac{1}{2} = \frac{2}{8}$